National University of Science and Technology
Department of Applied Mathematics

SMA 5273 OPERATIONS RESEARCH TECHNIQUES FOR MANAGEMENT

December 2005

Time: 3hours

Answer **ALL** questions in section A and any **THREE** from section B

Section A [40 marks]
Answer **ALL** questions from this section

A1. Truckco is trying to determine where they should locate a single warehouse. The positions in the $x-y$ plane (in kilometres) of their four customers and the number of shipments made annually to each customer are given in the table below.

Customer	X – Coordinate	Y - Coordinate	Number of Shipments
1	5	10	200
2	10	5	150
3	0	12	200
4	12	0	300

Truckco wants to locate the warehouse to minimize the total distance trucks must travel annually from the warehouse to the four customers. Formulate a NLP model.
[9 marks]

A2. A construction contractor estimates that the size of the workforce needed over the next 5 weeks to be 5, 7, 8, 4 and 6 workers respectively. Excess labour kept on the force will cost $300 000 per worker per week and new hiring in any week will incur a fixed cost of $400 000 plus $200 000 per worker per week. Determine an optimum plan for the contractor. **[12 marks]**

A3. Prove that if S is a convex set, $f(x): S \to \Re$ is a convex function, and $\bar{x}$ is a local mimimum of

$Min\ \ Z = f(x)$
$s.t.\ \ x \in S$
then $\bar{x}$ is a global minimum of $f(x)$ over S. **[9 marks]**

A4. (a) State a sufficient condition for a stationary point $x_o \in \Re^n$ to be an extremum.

(b) A monopolist producing a single product has two types of customers. If q_1 units are produced for customer 1, then customer 1 is willing to pay a price of $70 - 4q_1$ dollars. If q_2 units are produced for customer 2, then customer 2 is willing to pay a price of $150 - 15q_2$ dollars. For $q > 0$, the cost of manufacturing q units is $100 + 15q$ dollars. How many units should the monopolist sells to each customer so that he maximizes profit? **[10 marks]**

Section B [60 marks]

Answer any **THREE** questions from this section

B5. Mr. Murambatsvina, proprietor of the Murambatsvina Frame Company has a contract to be completed over four weeks which involves delivery of completed door and window frames to a new housing complex to satisfy the builder's schedule. The builder, Mr Hlalani requires 400 sets of frames in week 1, 700 sets in week 2, 900 sets in week 3 and 1300 sets in week 4 (Each set contains 3 door frames and 8 window frames). The frames cannot be delivered early so any storage must be at Murambatsvina's own premises which only has 1000 cubic metres of storage space which are needed for work-in-progress. The owner of the adjacent factory (which pickles onions) has some free space which he is willing to let Mr. Murambatsvina use, but he will have to hire an extra worker to transfer the frame sets into storage at a cost of $60 000 per hour. Mr. Murambatsvina estimates that the worker could store the sets at a rate of 25 per hour. The Murambatsvina factory can produce 600 frames per week using existing staff without incurring any extra costs. Output can be increased by overtime which costs an extra C dollars per week for extra 100 frame sets produced, where the cost varies over four weeks due to staff availability, C = $450000 in week 1, $400000 in week 2, $500000 in week 3 and $700000 in week 4. Find an optimal production schedule so that the builder's requirements are satisfied and Mr. Murambatsvina's total costs are minimized. **[20 marks]**

B6. (a) The Suny Corporation, a manufacturer of VCRs (video cassette recorders), wants to plan its production and inventory levels for the next six months. Its demand forecasts for the next six months are given in the second column of the table below:

Month	Demand Forecast	Unit Production Cost	Maximum Production Level	Minimum Production Level	Maximum Inventory Level	Minimum Inventory Level

1	1000	$460	7000	3500	7000	2500
2	4000	$470	5000	2500	7000	2500
3	6000	$480	4000	2000	7000	2500
4	5000	$500	8000	4000	7000	2500
5	3000	$500	6000	3000	7000	2500
6	2000	$490	3000	1500	7000	2500

Suny desires a plan that satisfies this demand with no backlogging; that is all demand must be met in the month it occurs.
Owing to fluctuations in the costs of such things as raw materials and utilities, a VCR's unit cost will vary from month to month. The third column of the table above specifies Suny's forecasts of each month's unit production costs.
Suny's maximum production level also fluctuates from month to month, owing to such things as differences in each month's required maintenance and the number of working days. It is Suny's policy not to change its workforce size from month to month. Consequently to prevent excessive idleness, Suny has set the minimum monthly production level at 50% of the maximum monthly production level. The table above's fourth and fifth columns contain each month's maximum and minimum production levels, respectively.
Suny currently has 3500 VCRs in inventory. To ensure sufficient safety stock, Suny has specified 2500 VCRs as the minimum permissible inventory level at the end of any month. Given the available storage space, Suny's maximum permissible inventory level at the end of any month is 700 VCRs. The table above's sixth and seventh columns contain each month's maximum and minimum inventory levels respectively.
Suny's accounting department has estimated that it costs $8 per month to hold a VCR in inventory, including the opportunity cost of foregone interest. Furthermore the accounting department recommends that Suny compute each month's inventory costs by multiplying the $8 per month figure by the average of the month's starting and ending inventory levels.
Given the above data, Suny's goal is to find production and inventory levels for each month that will minimize the total cost (total production costs plus total inventory costs) of satisfying forecasted demand with no backlogging. Formulate a bounded variable LP model. **[10 marks]**

(b) Prove that the optimum solution of the linear programming model

Min $z = CX$

s.t. $(A, I)X = b$

and $X \geq 0$

when finite occurs at an extreme point of the solution space. **[10 marks]**

B7. Consider the following linearly constrained optimization problem

Max $f(x_1, x_2) = \ln(1 + x_1) + x_2$

s.t. $2x_1 + x_2 \leq 3$

and $x_1, x_2 \geq 0$

where $\ln$ denotes natural logarithm.

(a) Use the KKT conditions to derive an optimal solution. **[11 marks]**

(b) Starting from the initial trial solution $(x_1, x_2) = (0,0)$, use one iteration of the Frank-Wolfe algorithm to obtain exactly the same solution you found in part (a) and then use a second iteration to verify that it is an optimal solution (because it is replicated). **[9 marks]**

B8. (a) Prove that $Q = \{X \mid (A,I)X = b, X \geq 0\}$ the set of all feasible solutions of the linear program is convex. **[6 marks]**

(b) The Cobb-Douglas production function for a particular manufacturer is given by $f(x_1,x_2) = 100x_1^{0.75}x_2^{0.25}$, where x_1 represents the units of labour at $150 000 per unit and x_2 represents the units of capital at $250 000 per unit. The total cost of labour and capital is limited to $50 000 000. Find the maximum production level for this manufacturer. **[8 marks]**

(c) The profit obtained by producing x_1 units of product A and x_2 units of product B is approximated by the model $f(x_1,x_2) = 8x_1 + 10x_2 - (0.001)(x_1^2 + x_1x_2 + x_2^2) - 10000$. Find the production level that produces a maximum profit. **[6 marks]**

END OF EXAMINATION