NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

MAPH 5236 - GEOPHYSICAL INVERSE THEORY

MSc GEOPHYSICS PART I: MAY 2005 *DURATION: 4 HOURS*

*Answer **all** questions. The maximum possible mark is 100 points*

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1. a) Compute the pseudo-inverse and resolution matrix of $\begin{pmatrix} 1 & -1 \\ 4 & -4 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$ [5]

   Assuming the right hand-side is $\begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}^r$, what is the least squares estimator of the 4-dimensional model vector ? [5]

   b) Assuming the data covariance matrix is $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.0001 \end{pmatrix}$, compute the covariance of the matrix of the least squares estimator. [5]

2. Given the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

   a) find the eigenvalues of $B^T B$ [5]
   b) compute its resolution matrix . [8]
   c) show that $B^T B + \lambda I$ is an invertible matrix [5]
   d) hence, determine the SVD of this matrix [10]

3. a) i) Explain what resolution is all about. [2]

   ii) Briefly explain the complications / difficulties encountered in resolution [6]

   b) Assume a linear problem with uncertainties only arising from random noise in the data. Write down the true earth model related to the observed data. Explain the relation you have written with reference to the true earth model and the generalised inverse solution. [8]

c) Suppose in (a) above, the resolution matrix is given by the expression

$$d^{+} = U_r U_r^T d$$

then the matrix $U_r U_r^T$ tells how well the data are predicted by the computed model. Explain. [5]

4. $d_i = m_1 + m_2 x_i + m_3 y_i$ where x and y are two auxiliary variables. Given the observed data $d_i = [10,-6,12,21,6]$; and $x = [1,-1,2,4,3]$; for $y = [2,-3,3,6,1]$ compute the least squares solution. [10]

5. Describe appropriately, the ***F test of error improvement significance***, explaining both its application and importance. [8]

6. Assume that the travel time $T_i(m) = T(m, x_i, y_i, z_i)$ to a receiver at (x_i, y_i, z_i) for a P and S wave from an earthquake at $m^T = (x_0, y_0, z_0, t_0)$ is calculable. Formulate the model for the P and S travel times for the arrival time t_i of a given wave at the i^{th} receiver. [6]

Comment on the relations formulated. [4]

The iterative least squares approach may be used to solve this problem. Derivatives ∇T_i must be computed,write down the required four finite difference formulae. In order to numerically calculate the derivative. [8]

END OF EXAMINATION