NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

MAPH 5236 – GEOPHYSICAL INVERSE THEORY

MSc IN GEOPHYSIC I: *MAY 2006* *DURATION: 4 HOURS*

ANSWER ALL QUESTIONS

1. Explain clearly, the terms;
   (a) expectation value, [3]
   (b) variance, [3]
   (c) bias and [3]
   (d) mean – squared error. [3]

2. Given two matrices A = $\begin{bmatrix} 1 & 2 \\ -3 & -2 \end{bmatrix}$ and B= $\begin{bmatrix} 0 & 6 \\ 1 & 1 \end{bmatrix}$

   (i) Show that $(AB)^T = B^T A^T$ and that [2]

   (ii) $(AB)_{ij} \neq (BA)_{ij}$ [2]

   (iii) Calculate the eigen vectors for matrix A and determine the eigen values for matrix B. [3], [3]

3. (a) (i) What do you understand by the term "singular value decomposition" (SVD)? [2]

   (ii) Given a matrix C = $\begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix}$, determine the SVD for C. [10]

   (b) Determine the column space and the null space of the two matrices.

   X= $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ and Y = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. [2], [2]; [2], [2]

   (c) Show that B=$\left(A^T A\right)^{-1} A^T$ is a left inverse and C= $A^T\left(AA^T\right)^{-1}$ is a right inverse of a matrix A, provided that AA^T and $A^T A$ are invertible? [4]
   When are $A^T A$ and AA^T invertible? [1]

4. Given a matrix G = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and that the trace of this matrix is $a + d$ and the determinant is $ad - cb$, show by direct calculation that the product of the eigen values is equal to the determinant and the sum of the eigen values is equal to the trace. [8]

5. (a) Calculate the SVD of the matrix A = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ directly by computing the eigen vectors of $A^T A$ and AA^T. Show that the pseudo inverse solution to the linear system $Ax = y$ where $y = (1,\ 2,\ 1)^T$ is given by averaging the equations. [9]

(b) Given the matrix B = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

(i) determine the generalized inverse of matrix B, [5]
(ii) determine the resolution matrix of B, and [5]
(iii) show that $B^T B + \lambda I$ is always an invertible matrix. [3]

6. Given a model,

$$d_i = m_1 + m_2 x_i + m_3 y_i \ ,$$

calculate the least squares solution for

d = [15, 0, 19, 18, 18, 5, -10, -7],
x = [2, 1, 4, 5, 3, -2, -1, -4],
y = [4, -2, 3, 1, 4, 5, -4, 2],

$m_1 = 1$, $m_2 = 3$ and $m_3 = 2$. [10]

7. A wheel has four numbers on it: 1, 2, 3, and 4. The even numbers are white while the odd ones are black. When the wheel is span twice, the following associated sample space is obtained:

$$[(1,1),(1,2),(1,3),(1,4)]$$
$$[(2,1),(2,2),(2,3),(2,4)]$$
$$[(3,1),(3,2),(3,3),(3,4)]$$
$$[(4,1),(4,2),(4,3),(4,4)]$$

Suppose A is the event that the first number is white and B is that the second event is white:

(i) what is the probability P(AB), that both numbers are white? [2]

(ii) Calculate the conditional probability, P(B|A). [3]

8. Given a set of values $A = (a_1, a_2, a_{3,}............................a_n)$ and $B = (b_1, b_2, b_3, ...b_n)$; write a short program in Fortran language that will read both A and B from two given files A and B, calculate the sum of the squares of A and B, and determine the co – variance of the data. Assume n runs from 1 to 60. The output should tabulate the values of A, B and the sum of the squares as well as give the value of the co-variance at the end. [8]

END OF EXAMINATION