# NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

## APPLIED PHYSICS DEPARTMENT

## SPH 1201 – WAVES AND OPTICS I

BSC HONOURS PART I : MAY 2002 DURATION : 3 HOURS

**ANSWER ALL QUESTIONS IN SECTION A AND ANY THREE QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.**

| | | |
|---|---|---|
| Atomic mass of oxygen | = | 16 *a.m.u.* |
| Atomic mass of hydrogen | = | 1 *a.m.u.* |
| Speed of sound in air | = | 341 $ms^{-1}$ |
| 1 a. m. u. | = | $1.66 \times 10^{-27}$ kg |

## SECTION A

1. (a) Two identical masses of two pendula are coupled by a weightless spring of spring constant $k$. The system is set into vibrational motion along the line of the spring.

   (i) State the possible normal modes of vibration for the system and [2]

   (ii) write the equations of motion for each mass. [2]

   (b) Illustrate graphically the difference between (a) linear restoring force (b) non-linear 'hard' spring (c) non-linear 'soft' spring. [4]

   (c) Distinguish between 'phase velocity' and 'group velocity' in wave motion. Include a mathematical expression for each in your answer. [4]

   (d) Show that $y = y_m \sin(kx - wt)$ may be written in the alternative forms

   $y = y_m \sin k(x - vt)$, $\quad y = y_m \sin [\, 2\pi (x - vt)/\lambda \,]$

   $y = y_m \sin w(x/v - t)$ $\quad y = y_m \sin 2\pi (x/\lambda - t/T)$ [4]

(e) Explain the term *superposition*. Give at least two real life examples where superposition occurs. Under what conditions does the principle of superposition fail? [4]

(f) Express in dB the difference in the intensity of two sound waves, if the pressure amplitude of one is twice that of the other. [4]

(g) State one application of ultrasound in medical physics. Give merits and demerits of ultrasound over x-rays. [4]

(h) Explain the conditions under which constructive interference between two waves of the same frequency $w$ can occur. Explain your answer. [4]

(i) Is Young's experiment an interference experiment or a diffraction experiment? Explain. [4]

(j) Explain what is meant by plane polarised light and briefly describe two methods by which it can be produced. [4]

**SECTION B**

2. Consider a linear array of 2N particles of alternating masses $m_1$ and $m_2$.

0, 1, 2, 3, 4, 5, .......... 2N-1, 2N, 2N+1

The transverse displacement of the $p^{th}$ particle is labeled $y_p$. We can label all p values with the index r, 1,r,N, by setting p=2r for even values of p and p=2r-1 for odd values of p.

The equations of motion for the $2r^{th}$ and $2r+1^{th}$ particles are

$$m_2\ddot{y}_{2r} + \frac{T}{l}(y_{2r} - y_{2r-1}) - \frac{T}{l}(y_{2r+1} - y_{2r}) = 0$$

$$m_1\ddot{y}_{2r+1} + \frac{T}{l}(y_{2r+1} - y_{2r}) - \frac{T}{l}(y_{2r+2} - y_{2r+1}) = 0$$

Assume that each displacement is of the form $y_p = A_p\cos(\omega t)$.

(a) Show that the equations of motion together with the solutions for $y_p$ place the following restrictions on the amplitudes $A_p$

$$\frac{A_{2r+1} + A_{2r-1}}{A_{2r}} = \frac{2\omega_{02}^2 - \omega^2}{\omega_{02}^2}$$

and

$$\frac{A_{2r+2} + A_{2r}}{A_{2r+1}} = \frac{2\omega_{01}^2 - \omega^2}{\omega_{01}^2}$$ [8]

with $\omega_{01}^2 = T/(m_1 l)$ and $\omega_{02}^2 = T/(m_2 l)$.

(b) If the amplitudes $A_{2r}$ and $A_{2r+1}$ can be written as

$A_{2r} = C\sin(2r\theta)$

$A_{2r+1} = D\sin((2r+1)\theta)$,

find the allowed values of $\theta$. [4]

(c) Using the amplitudes in b) and the relationships from a) show that the mode frequencies can be determined from the expressions

$$\left(2\omega_{02}^2 - \omega^2\right)C - 2\omega_{02}^2 \cos\left(\frac{n\pi}{2N+1}\right)D = 0$$

$$-2\omega_{01}^2 \cos\left(\frac{n\pi}{2N+1}\right)C + \left(2\omega_{01}^2 - \omega^2\right)D = 0$$ [8]

3. (a) (i) Write down the equation governing the propagation of a longitudinal wave in a gaseous column, explaining the notations used therein. [2]

(ii) Show that the pressure variations also obey the wave equation. [5]

(b) Sound travels through gases with speed $c = (B/\rho)^{1/2}$, where B is the Bulk modulus of the gaseous system and $\rho$ is its density.
Compare the velocities of sound propagation at $0^{\circ}$C in oxygen and in hydrogen.
[NOTE: Consider the process to be adiabatic with $\gamma = C_p/C_v = 1.40$ for both the gases] [7]

(c) Two car sirens C and D each have a frequency of 600Hz. C is moving to the left away from D with a velocity of 80km/h, whilst D is stationary. An observer is between the two sirens, moving to the left with velocity 10 m/s.

(i) What frequency does the observer hear from siren C? [2]
(ii) What frequency does the observer hear from siren D? [2]
(iii) What is the beat frequency? [2]

4. (a) Considering a 2D wave in an environment of rigid boundaries, explain the principle of wave-guides. [6]

(b) Explain briefly how the principle of wave guide is used in optical fibres. Discuss the process by which light is bent around corners in a fibre. [6]

(c) Mention two advantages and two disadvantages of the use of optical fibres. Give two applications of optical fibre. [8]

5. Explain the formation of Newton's rings and comment on the colour pattern of the fringes when white light is used. [5]

Figure 1 shows a lens of radius of curvature R resting on a glass plate and illuminated from above by light of wavelength $\lambda$. Circular interference fringes appear.

(a) Find the radii of the circular interference maxima assuming that $r/R << 1$. [5]

(b) The lens in the Figure 1 is illuminated with sodium light incident normally; if the diameter of the tenth black ring is 1.5cm, find the curvature of the lens surface(wavelength = 589 nm). [5]

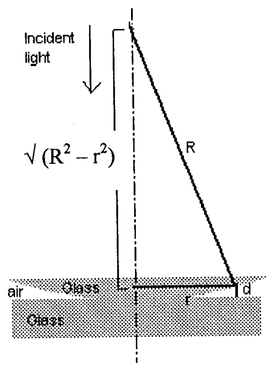


Figure 1

(c) Explain what is meant by Newton's scale of colours? [5]

6. (a) Explain and illustrate the difference between Fresnel and Fraunhofer diffractions. [6]

(b) A slit of width $a$ is illuminated by white light.

(i) For what value of $a$, will the first minimum for red light ($\lambda = 650$nm) fall at $\theta = 30^{\circ}$. [3]

(ii) What is the ratio of the slit width to wavelength for this case? [3]

(c) In (b) what is the wavelength $\lambda'$ of the light whose first diffraction maximum (excluding the central maximum) falls at $\theta = 30^{\circ}$, thus coinciding with the first minimum for the red light? [4]

(d) What requirements must be met for the central maximum of the envelope of the double slit Fraunhofer pattern to contain exactly eleven fringes? [4]

**- END OF PAPER –**