## SPH 1201 - WAVES AND OPTICS

BSc HONOURS PART I: DECEMBER 2002 DURATION: 3 HOURS
ANSWER ALL PARTS OF QUESTION 1-IN SECTION A AND ANY THREE QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B 60 MARKS

Speed of sound (at $20^{\circ} \mathrm{C}$ )
Speed of Electromagnetic wave
$343 \mathrm{~m} / \mathrm{s}$
$3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$

## SECTION A

1. (a) Give an expression for the total energy in a coupled pendulum and define all the quantities and terms involved.
(b) A certain bat produces a sound wave with wavelength of 34 mm . Can a human being hear this sound? Explain.
(c) Give the importance of Fourier analysis in a wave motion. Include a graphical illustration.
(d) A plane sinusoidal acoustic wave falls normally on a quiet air-water surface. Assuming no energy losses at the boundary, find:
(i) the sound power reflection coefficient and
(ii) the sound power transmission coefficient
(e) The electric field of a plane electromagnetic wave is given as:
$\mathrm{E}_{\mathrm{x}}=0$
$\mathrm{E}_{\mathrm{y}}=0.5 \cos \left[2 \pi \times 10^{8}(t-x / c)\right] \quad N / C$
$\mathrm{E}_{\mathrm{z}}=0$
What is the wavelength and the direction of propagation of this wave?
Explain.
(f) Explain how total internal reflection (TIR) is applied in optical fibres. Give two conditions necessary for TIR to occur.
(g) Illustrate graphically the difference between, non-dispersive, normal dispersive and anomalous dispersive, media.
(h) Comment on the phase difference between successive fringes in Fraunhoffer and Fresnel's diffraction patterns.
(i) What is the difference between randomly polarised light and plane polarised light?

## SECTION B

2. $\quad \mathrm{A} \mathrm{H}_{2} \mathrm{O}$ molecule can be modelled as a system consisting of a central mass, $\mathrm{m}_{2},\left(\mathrm{O}^{16}\right.$ atom) connected by springs on either side to masses, $m_{1}$ and $m_{3}$ ( $\mathrm{H}^{1}$ atoms).
(a) Construct and solve the equations of motion for each of the masses for the two normal modes in which the masses oscillate along the line joining them. [12]
(b) Using the fact that $\mathrm{m}_{1}=\mathrm{m}_{3}=1 \mathrm{amu}$ and $\mathrm{m}_{2}=16 \mathrm{amu}$, determine the ratio of frequencies of the two modes.
3. (a) Comment briefly on the effect of a symmetric non-linear return force on the vibrational motion of a free undamped oscillator.
(b) A system shown in the figure is set into motion under the action of a driving force, $F_{0} \cos \omega \alpha$. Derive the equation of motion for the system given that the. tension in the string is $T=T_{0}+s(L-a)$

(c) By assuming a first approximation solution of $\mathrm{x}_{1}=\mathrm{A} \cos \mathrm{wt}$, find the second approximation solution.
[NOTE: $\quad \cos ^{3} \omega t=\frac{3}{4} \cos \omega t+\frac{1}{4} \cos 3 \omega t$ ]
4. (a) Write down the wave equation governing the propagation of a disturbance along a wave guide. Define the terms appearing in this equation.
(b) The displacement of the waves on a waveguide of width, b is given by $z=A \sin \left[\omega t-\left(k_{1} x+k_{2} y\right)\right]+A_{2} \sin \left[\omega t-\left(k_{1} x-k_{2} y\right)\right]$ with boundary conditions, $\mathrm{z}=0$ at $\mathrm{y}=0$ and at $\mathrm{y}=\mathrm{b}$. Show that $z=-2 A_{1} \sin k_{2} y \cos \left(\omega t-k_{1} x\right)$, and determine the expression for $k_{2}$. Explain the resultant equation, including how the sine term affects the amplitude of the wave.
(c) Show that the phase and group velocities for the equation obtained in part (b) are given by $V_{p}=\left(\frac{k}{k_{1}}\right) V$ and $V_{g}=\left(\frac{k_{1}}{k}\right) V$ respectively, where V is the velocity of light.
(d) Explain how the waveguide acts as a frequency filter.
5. (a) Explain how diffraction phenomena are used in distinguishing two distant stars that are at small angular separation. In your answer include an explanation of Rayleigh’s criterion.
(b) (i) What is a diffraction grating?
(ii) A parallel monochromatic beam of light is incident normally on a diffraction grating. Under what conditions will a maximum on a distant screen be observed?
(iii) What pattern is observed when the interference factor (due to double slit interference) is multiplied with the diffraction factor (due to diffraction from a single slit)? Explain.
6. (a) (i) Define the term random phase difference.
(ii) In which ranges of the electromagnetic wave is polarisation readily observed?
(b) How is an electromagnetic wave affected
(i) when it is transmitted through a polaroid and
(ii) when it is transmitted through two crossed polarising sheets?
(c) Two polarising sheets are set such that the intensity of the transmitted light is a maximum. Through what angle must either sheet be turned if the intensity is to drop by one half?
