

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 1201 – WAVES AND OPTICS

BSc HONOURS PART I: MAY 2005 DURATION: 3 HOURS

ANSWER ALL PARTS OF QUESTION ONE IN SECTION A AND ANY THREE QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS

SECTION A

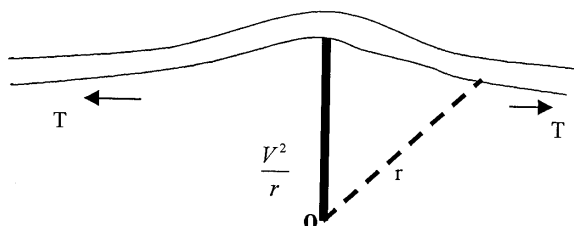
1. (a) (i) A simple harmonic oscillator has a period  $4\pi$ . It passes through a point  $0.39\text{m}$  with velocity  $3.9\text{m/s}$  away from origin  $0$ . How much time lapses before it next passes through this point? [2]
- (b) A body of mass  $2\text{kg}$  stretches a spring  $8\text{cm}$  when it hangs vertically in equilibrium. The body is attached to the spring on a frictionless horizontal surface, displaced  $4.5\text{cm}$  from equilibrium position and released.
- (i) Find its frequency: amplitude, period and phase constant  $\Phi$ . [4]  
(ii) What is the maximum velocity of the oscillator and when does it occur? [3]  
(iii) A seconds pendulum (whose  $T/2 = 1$  second at sea level) is raised to  $1\text{ km}$  above sea level. How many seconds will it loose in a day? [4]
- (c) The interatomic potential energy for a particular crystal is given by:
- $$V(r) = \frac{\beta^2}{4\pi\epsilon_0 r} \left[ \frac{1}{9} \left( \frac{r_0}{r} \right)^8 - 1 \right]$$
- where all the symbols have their usual meanings.
- (i) Find the anticollapse constant  $\beta$  in terms of  $r_0$ . [4]  
(ii) Prove that the rate of expansion of the crystal is a constant value. [5]
- (d) Two travelling waves moving in a common direction interfere with each other, amplitude of each is  $Y_m = 10.2\text{mm}$ . The phase difference  $\Phi$  between them is  $\Phi = 107^\circ$
- (i) What is the amplitude  $Y_m^1$  of the resultant wave? [2]  
(ii) What value of  $\Phi$  would lead to the resultant wave having the same amplitude as the amplitudes of the combining waves? [3]
- (e) (i) Differentiate between ultrasound and infrasound? Give one application of ultrasound. [3]  
(ii) The audible frequency range for normal hearing is from about  $20\text{Hz}$  to  $20\text{KHz}$ . If speed of sound in air ( $V$ ) is  $343\text{m/s}$ , what are the wavelengths of sound waves at these frequencies? [2]

(iii) Given the above speed for sound, what are the fundamental and first order frequencies and their wavelengths for standing waves in closed tube 1m long [4]

(f) Comment on the phase difference between successive fringes in Fraunhofer and fresnel diffraction patterns. [4]

2. The diagram above shows a string wave pulse where T is the string tension and V is the pulse velocity, r is the radius of the circular arc.

(a)



(i) Show that the velocity of the pulse depend on the properties of the medium and is given by:

$$V = \sqrt{\frac{T}{\mu}} \text{ where } \mu \text{ is the mass linear density of the string. [7]}$$

(ii) State one important assumption made to up with above relation. [2]

(b) (i) Show that for an ideal gas, the velocity of a sound wave travelling through it at under isothermal conditions is in terms of pressure given by:

$$V = \sqrt{\frac{P}{\rho}} \text{ where } P \text{ is pressure and } \rho \text{ is density. [7]}$$

(ii) Show that in terms of temperature, the above relation becomes

$$V = \sqrt{\frac{RT}{M}} \text{ where } R \text{ is the universal gas constant, } m \text{ is the molecular mass. [2]}$$

(iii) The two relations above introduce an error in V. Explain why and how this error should be corrected. [2]

3. (a) Two car sirens A and B have each a frequency of 500Hz. A is moving to the left away from B with a velocity 50km/h whilst B is stationary. An observer is between the two sirens, moving to the left with velocity of 6m/s.

(i) What frequency does the observer hear from sirens A? [2]

(ii) What frequency does the observer hear from siren B? [2]

(iii) What is the beat frequency in this case? [2]

(b) Show that the following shifts occur in the frequencies detected by the receiver:

(i)  $f^1 = f \frac{V}{V \pm U_s}$  when receiver is stationary and source is moving. [5]

(ii)  $f^1 = f \frac{V \pm U_D}{V}$  when receiver is moving and source is stationary. [5]

where:  $f$  is source frequency  
 $f^1$  is the received frequency  
 $V$  is the wave velocity  
 $U_s$  is velocity of source  
 $U_D$  is velocity of detector

- (c) Show that general Doppler effect equation, that is the one in which both source and receiver are moving with respect to the air mass between them is:

$$f^1 = f \frac{V \pm U_D}{V \pm U_s} \text{ where all symbols have their usual meanings.}$$

4. (a) For a stretched string wave, the relation between the displacement,  $Y$ , of any element at position  $X$  at a time  $t$  is given by

$$Y(x,t) = Y_m \sin(kx - \omega t). \text{ Use this relation to define}$$

- (i) wave number ( $K$ ) [2]  
(ii) period of oscillation ( $T$ ) [2]

- (b) A travelling sinusoidal wave is described by:

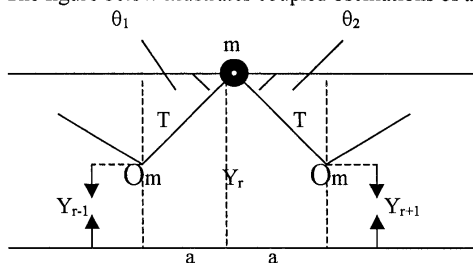
$$y(x,t) = 0,04 \sin(74.1x - 2,4t), \text{ constants are in S.I units}$$

- (i) What is the amplitude of this wave? [1]  
(ii) At  $t = 0s$ , what are the displacements of particles at  $X = 0.0m$ ;  $0.2m$  and  $0.5m$ ? [3]  
(iii) At  $x = 0.1m$ , what is the displacement at  $t = 0.15$  and  $0.2s$ ? [1]  
(iv) What is the maximum velocity of oscillation of the particles of transmitting medium? [3]  
(v) What is the velocity of propagation of the wave in the string [3]
- (c) A particle is under the influence of two combined simple harmonic motions along the same line, their equations are:

$$Y_1 = Y_m \sin(Kx - \omega t + \Phi) \text{ and } Y_2 = Y_m \sin(Kx - \omega t)$$

Find the resultant equation of motion of the particle and its amplitude. [5]

5. The figure below illustrates coupled oscillations of a loaded string.



$n$  equal masses are spaced at equal distances,  $a$

- (i) Show that the equation of motion for the  $r^{\text{th}}$  mass is given by:

$$\ddot{y}_r = \frac{T}{ma}(y_{r-1} + y_{r+1} - 2y_r) \quad [5]$$

- (ii) By considering  $y_r = A\gamma e^{i\omega t}$ , show that the above equation can be reduced to:

$$-A_{r-1} + \left(2 - \frac{maw^2}{T}\right)A_r - A_{r+1} = 0 \quad [5]$$

- (iii) Show how equation (i) by considering appropriate limits, can be used to derive the Wave Equation. [7]

- (iv) Show that the dimensions of  $\frac{P}{T}$  in the Wave Equation are similar to those of  $\frac{1}{V^2}$ . [3]

6. (a) An interference pattern is observed on a screen placed a distance  $d$  from two light sources  $S_1$  and  $S_2$  of separation  $a$  when monochromatic light of wavelength  $\lambda$  is falling normally on the plane of the sources. Describe the observed pattern and state the conditions for constructive/destructive interference. [6]

- (b) Show that the intensity of the resultant motion at any point on the screen is given by:

$$I = I_0 \cos^2(\pi ax / \lambda d)$$

where  $I_0$  is none intensity distribution [8]

- (c) Explain how Michelson's interferometer can be used to define the metre. [6]

END OF EXAM

