NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 1201 - WAVES AND OPTICS

BSc HONOURS PART I: *MAY 2006* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF SECTION A AND ANY **THREE** QUESTIONS IN SECTION B.
SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS

Atomic mass of oxygen	=	16 a.m.u
Atomic mass of hydrogen	=	1 a.m.u
Speed of sound in air	=	$341ms^{-1}$
1 a.m.u	=	1.66×10^{-27}kg

SECTION A

1. (a) Define the following terms as used in wave motion:
   (i) particle velocity, [2]

   (ii) phase velocity, [2]

   (iii) group velocity. [2]

   (b) (i) A simple harmonic oscillator has a period 4π. It passes through a point 0.38m away from origin O, with velocity $3.8ms^{-1}$. How much time elapses before it passes through this point again? [3]

   (ii) The atoms of a solid at room temperature vibrate at a frequency of about 10^{13}Hz and with amplitude of 10^{-11}m. If the mass of an atom is 10^{-25}kg and the motion is approximately simple harmonic, find the maximum kinetic energy, the maximum acceleration of an atom as well as the value of the force constant for the motion. [4]

   (c) For a stretched string wave, the relation between the displacement, y, of any string element at position x at a time t is given by

$$y(x,t) = y_m \sin(kx - \omega t).$$

   Use this relation to define the following quantities:
   (i) wave number (k), [3]

   (ii) period of oscillation (T). [3]

(d) (i) Explain the difference between *infrasound* and *ultrasound* and list one application of each. [5]

(ii) The inter atomic potential energy for a particular crystal is given by:

$$V_r = \frac{e^2}{4\pi\varepsilon_o r}\left[\frac{1}{9}\left(\frac{r_o}{r}\right)^8 - 1\right]$$

where all the symbols have their usual meanings. Find an expression for the anticollapse constant for such a crystal in terms of r_o [6]

(e) Explain the application of total internal reflection in optical fibres. [3]

(f) Give two conditions necessary for total internal reflection to occur. [2]

(g) Given that the speed of sound in a medium is $343ms^{-1}$, what are the fundamental and first order frequencies and their wavelengths for standing waves in a 2m long closed tube. [5]

SECTION B

2. (a) (i) Write down the equation governing the propagation of a longitudinal wave in a gaseous column, explaining the notation used. [2]

(ii) Show that the pressure variations also obey the wave equation. [5]

(iii) Show that for the phenomenon of Doppler effect, $f' = f\left(1 \pm \frac{V}{v}\right)$ where:

f is the source frequency
f' is the modified frequency
V $= |(V_S \pm V_D)|$, V_S and V_D are source and detector velocities.
v is the wave velocity
for the condition V_S and V_D are very small compared to v. [7]

(b) Two car sirens A and B each have a frequency of 500Hz. A is moving to the left away from B with a velocity 50km/h, whilst B is stationary. An observer is between the two sirens, moving to the left with velocity = 6m/s.
(i) what frequency does the observer hear from siren A? [2]
(ii) what frequency does the observer hear from siren B? [2]
(iii) what is the beat frequency? [2]

3. (a) Briefly give the meanings of the following:
(i) linear restoring force,
(ii) non-linear "hard" spring,
(iii) non - linear "soft" spring.
Use a graph for your illustrations. [5]

(b) The system shown below is set into motion under the action of a driving force given by, $F_o \cos\omega t$.

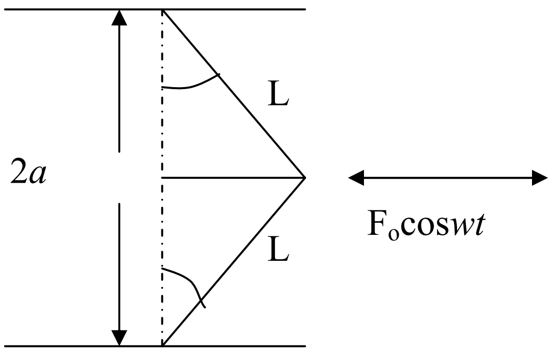

Figure 1.

The tension in the string is given by $T = T_o + s(L - a)$, where all symbols have their usual meanings. Derive the equation of motion for the system. [10]

(c) By assuming a first approximation solution of $x_1 = A \cos\omega t$, find the second approximation solution.
[NOTE: $\cos^3\omega t = \frac{3}{4} \cos \omega t + \frac{1}{4} \cos 3\omega t$]. [5]

4. (a) Explain the difference between Fresnel and Fraunhofer diffraction. [6]

(b) A slit of width a is illuminated by white light.
(i) For what value of a will the first minimum of red light $(\lambda = 650nm)$ fall at $\theta = 30^0$? [3]
(ii) What is the ratio of the slit width to wavelength for this case? [3]

(c) In (b) what is the wavelength λ of the light whose diffraction maximum (excluding the central maximum) falls at $\theta = 30^0$, thus coinciding with the first minimum for red light? [4]

(d) What requirements must be met for the central maximum of the envelope of the double slit Fraunhofer pattern to contain exactly 11 fringes? [4]

5. (a) An interference pattern is observed on a screen placed a distance d from two light sources S_1 and S_2 of separation a when monochromatic light of wavelength λ is falling normally on the plane of the sources. Describe the observed pattern and state the conditions for constructive/destructive interference. [6]

(b) Show that the intensity of the resultant motion at any point on the screen is given by:

$$I = I_o \cos^2(\pi a x/\lambda d)$$

where I_o is the intensity of the centre point. [8]

(c) Explain how the Michelson's interferometer can be used to define the metre. [6]

6. The figure below illustrates coupled oscillations of a loaded string.

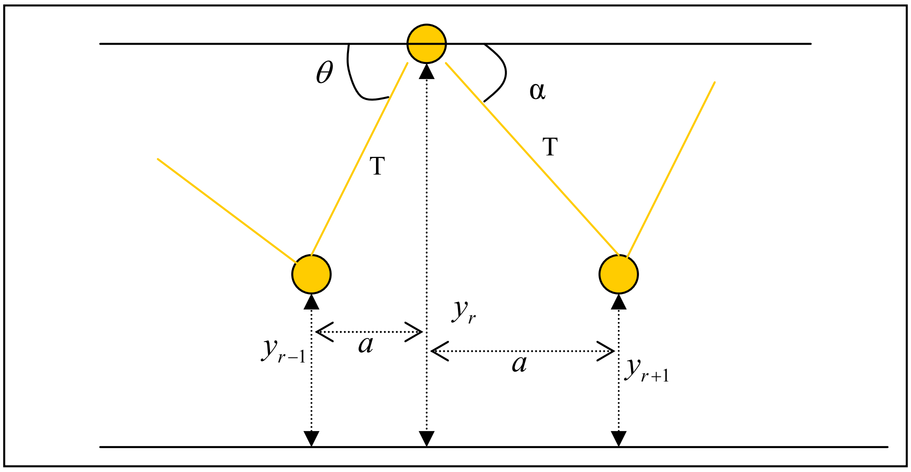

Figure 2.

n equal masses (*m*) are spaced at equal distances (*a*),

(i) show that the equation of motion for the r^{th} mass is given by

$$\left(\frac{d^2 y_r}{dt^2}\right) = \frac{T}{m_a}\left(y_{r-1} + y_{r+1} - 2y_r\right)$$ [5]

(ii) By considering $y_r = Are^{i\omega t}$, show that this equation can be reduced to:

$$-Ar - 1 + \left(2 - \frac{ma\omega^2}{T}\right)Ar - Ar + 1 = 0$$ [5]

(iii) By considering appropriate limits show how equation (i) can be used to derive the *wave equation.* [7]

(iv) Show that the dimensions of $\frac{\rho}{T}$ are similar to those of $\frac{1}{v^2}$. [3]

- END OF EXAMINATION -