NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHSYICS DEPARTMENT

SPH 2101 - QUANTUM MECHANICS

BSc HONOURS PART II: *JANUARY 2003* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF QUESTION 1-IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B 60 MARKS

SECTION A

1. (a) (i) Sketch the potential that an electron feels inside the Cesium if work function of Cesium is 1.9eV [4]

(ii) From (i) calculate the maximum kinetic energy with which an electron can Escape from Cesium when light of $\lambda = 4000\overset{o}{A}$ shines onto it? [4]

(b) A particle is confined in a one dimensional box of length a. Given that the n[th] normalised eigen functions are:

$$\varphi_n(x) = A\cos\left(\beta_n x - \frac{\pi}{2}\right) \text{ where n = 1, 2, 3, 4, ... n.}$$

(i) Give expressions for A and β_n in terms of a and n. [6]

(ii) Calculate the expectation value of the kinetic energy. [4]

(c) (i) What are the possible values of J for f states of a hydrogen atom? [4]

(ii) What are the corresponding m_j values? [3]

(iii) How many total m_j states are there? [3]

(d) A particle of mass m is confined in a one – dimensional potential well. The potential is:

$V(x) = -V_0, \quad 0 \le x \le a$ and

$V(x) = 0 \quad x < 0$ and $x > a$

Suppose that at a time $t = 0$ the state function describing the motion of the particle in the potential is;

$$\psi(x,0) = \frac{2b}{a}x; \quad \text{for} \quad 0 \le x \le \frac{a}{2} \quad \text{and}$$

$$\psi(x,0) = 2b\left(\frac{x}{a} - 1\right); \quad \text{for} \quad \frac{a}{2} \le x < a$$

(i) sketch the potential and the state function, [4]
(ii) determine b so that $\psi(x,0)$ is normalised. [5]

(e) Explain the spin-orbit interaction. [3]

SECTION B

2. A particle exists in the n^{st} excited state level of a one dimensional infinite potential box with width, L. From the solution for the wave function,

(i) find the expectation values of x, x^2, p_x, p_x^2. [12]
(ii) In terms of the quantities found above, define Δx and Δp_x , [3]
(iii) hence find the value of the product $\Delta x \Delta p_x$. [3]
(iv) Find and $[p_x, x]$. [2]

3. A particle is subjected to following potential:

$V(x) = \infty$ for $x < 0$,
$V(x) = -V_0$ for $0 < x < a$,
$V(x) = 0$ for $x > a$,

V_0 is a positive quantity.

(a) Sketch the potential. [4]

(b) Solve the Schroedinger equation for energies E > 0. Evaluate all undetermined coefficients in terms of a single coefficient. [*You do not need to normalise the wave function*]. [8]

(c) Sketch the wave function in all the regions where the particle is likely to be found. [5]

(d) In a case where $E = -E_b$ where E_b is a positive quantity, sketch the wave function in the regions where the particle is likely to be found. [*You need not solve the Schroedinger equation*]. [3]

4. (a) Give an expression for the total energy of an electron in an hydrogen atom

in the presence of a uniform magnetic field, B. Identify the quantities that you use. [5]

(b) If the magnetic field strength varies with position in a certain direction, give an expression for the force that the electron feels. Identify and define all the quantities that are involved. [5]

(c) A beam of silver atoms with an average velocity of $7 \times 10^2 ms^{-1}$ passes through an inhomogeneous magnetic field 0.1m long which has a gradient of 3×10^2 Tm^{-1} in a direction perpendicular to the motion of the atoms. Find the maximum separation of the two beams which emerge from the field region. [*Assume that the net magnetic moment of each atom is 1.0 Bohr magneton*]. [10]

5. A particle of mass m moves in a rectangular box having dimensions a, b, c. Inside the box, the particle experiences zero potential; outside however the potential is infinite.

(a) Express V(x,y,z) in terms of x, y and z. [3]

(b) Determine the allowed energies for this system. [10]

(c) Write down the first three lowest energy levels for this particle. [3]

(d) Write down the wave functions for the first 3 degenerate states. [4]

6. (a) The state of a one-electron atom is given as $\psi_{n,l,m_l}(r,\theta,\varphi)$.

(i) Define the co-ordinates that form the argument of this wave function and the subscripts n, l, and m_l. State how n, l and m_l relate to r, θ and φ. [4]

(ii) Find the expectation value < V > for the potential energy in the ground state of the hydrogen atom and show that in the ground state the total energy $E = \frac{1}{2} < V >$ and hence find the kinetic energy in the same state.

The ground state wave function is

$$\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \exp\left\{-\frac{Zr}{a_0}\right\}$$ [6]

(b) (i) Deduce the equation of motion:

$\dot{A} = \frac{i}{\hbar}[H, A]$ where $\hat{H}$ is the Hamiltonian and $\hat{A}$ is not a function of time explicity. [6]

(ii) Given that the Hamiltonian for the Harmonic oscillator $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ show from the equation of motion deduced from (i) above, that the force F = -kx (as the case should be). [4]

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