NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 2101 - QUANTUM MECHANICS

BSc HONOURS PART II: *MAY 2005* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF QUESTION **ONE** IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.

SECTION A

1. (a) A nucleus of charge Z is bombarded by a proton and a neutron separately. Both have some kinetic energy as they approach the nucleus from a distance. Explain and compare what is likely to happen to each one of the particles at impact. [6]

(b) (i) What are the possible values of j for the d states in a one electron atom. [4]

(ii) What are the corresponding m_j values? [4]

(c) For the ground state of a simple harmonic oscillator given that the wave function is $\varphi_o(x) = A\exp(-\beta^2 x^2/2)$ where $\beta^2 = \frac{m\omega}{\hbar}$ and $\omega^2 = \frac{k}{m}$. Given that $\int_0^\infty x^2 e^{-ax^2}dx = \frac{1}{4a\sqrt{\frac{\pi}{a}}}$

evaluate

(i) A [3]

and

(ii) $< x >$ and $< p^2 >$ [5]

(d) Given that the one-dimensional Hamiltonian for a particle of mass m is $H = \frac{p_x^2}{2m} + V(x)$.

Evaluate the following commutators:

$[\hat{H},\hat{x}]$, $[\hat{H},\hat{p}_x]$ and $[\hat{p}_x,\hat{x}]$. [5]

(e) Given that the Hamiltonian for a Harmonic Oscillator $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$. From the equation of motion $\dot{A} = \frac{i}{\hbar}[\hat{H},\hat{A}]$ deduce the force on a particle executing this harmonic motion. [8]

(f) Which of the following functions are well behaved? For those functions that are not well behaved indicate the reason.

(i) $f(x) = Ae^{-\beta x}$, $C\sin(ax)$, $De^{-\alpha(x-\beta x^2)}$, $B\sinh(bx)$ and $\frac{\beta x^2}{(1-x^2)}$. [5]

SECTION B

2. Some nuclei are α- particle emitters. When the α- particle He^{++} is emitted it is influenced by two potentials - one in the nucleus and the other outside the nucleus.

(a) Name these potentials. [4]

(b) Sketch an approximate resultant potential, which should explain both α- emission and proton capture by the nucleus. [8]

(c) Explain nuclear fusion a process in which two nuclei coalesce to form one nucleus. Use a labelled diagram to give your explanation. [8]

3. (a) Name and explain an experiment in which space quantization is demonstrated. [6]

(b) When a hydrogen atom is exposed to an intense magnetic field, how would you express its total energy? [4]

(c) Derive an expression for the interaction energy of a Hydrogen atom in the presence of a strong magnetic field B and hence explain the splitting in the frequencies observed and the size of the splitting if the strength of the magnetic field is $B = 0.9T$. [10]

4. (a) Explain the fourth quantum number which is assigned to atoms. [4]

(b) Describe an experiment that demonstrates the existence of the spin magnetic moment in one-electron atoms. [6]

(c) Determine the gradient of a 50cm long Stern - Gerlach magnet that would produce a 1 mm separation at the end of the magnet between the two components of a beam of silver atoms emitted with typical kinetic energy of a 960°C oven. The dipole moment of the silver is due to the $l = 0$ electron just as in hydrogen. [10]

5. (a) Write down the full Hamiltonian of an electron of the Hydrogen atom in three dimension and Cartesian co-ordinates. [4]

(b) In order to solve the Schrödinger Equation one resorts to transformation of co-ordinates $(x, y, z) \rightarrow (r, \phi, \varphi)$. Express x, y and z in terms of r, θ and φ in this transformation. [4]

(c) The state of a one - electron atom in spherical co-ordinates (r,θ,φ) is given by $\psi_{n,l,m_l}(r, \theta, \varphi)$

(i) Define the subscripts $n,\ l,\ m_l$. [4]

(ii) Find the expectation value < V > for the potential energy in the ground state of the hydrogen and show that in the ground state the total energy $E = \frac{1}{2} < V >$. The ground state wave function is $\psi(r, \theta, \varphi) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_o}\right)^{3/2}$ x exp $\left\{-\frac{Zr}{a_o}\right\}$. [8]

6. (a) Show that the operator $\hat{L} = -i\hbar \bar{r}\, x \nabla$ is a Hermitian operator. [3]

(b) For a physical quantity A, $\dot{A}$ is defined to mean an operator whose expectation value in any state $\psi(r, t)$ is the time derivative of the expectation value of the operator $\hat{A}$. Show that $\dot{A} = \frac{i}{\hbar}[H, A]$, where H is the Hamiltonian of the system. [8]

(c) If the Hamiltonian operator of a system is $\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \hat{V}(x, y, z)$, using the equation in (b) give expressions for:

(i) the velocity components in the x, y, and z directions. [4]

(ii) the torque components in the x, y, z directions. [5]

- END OF EXAM -