NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHSYICS DEPARTMENT

SPH 2101 - QUANTUM MECHANICS

SUPPLEMENTARY EXAMINATION

BSc HONOURS PART II: *JULY 2005* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF QUESTION 1-IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B 60 MARKS

SECTION A

1. (a) (i) What is the maximum kinetic energy with which an electron can escape from Cesium (work function = 1.9eV) when light of $\lambda = 4000\overset{o}{A}$ shines onto it? [4]

(ii) Sketch the potential that an electron feels inside the Cesium. [4]

(b) A particle is confined in a one dimensional box of length a. Given that the nth normalised eigen functions are:

$$\varphi_n(x) = A\cos\left(\beta_n x - \frac{\pi}{2}\right) \quad \text{where n = 1, 2, 3, 4, ... n.}$$

(i) Give expressions for A and β_n in terms of a and n. [6]

(ii) Calculate the expectation value of the kinetic energy. [4]

(c) (i) What are the possible values of J for f states of a hydrogen atom? [4]

(ii) What are the corresponding m_j values? [3]

(iii) How many total m_j states are there? [3]

(d) A particle of mass m is confined in a one – dimensional potential well. The potential is:

$V(x) = - V_0, \quad 0 \le x \le a$ and

$V(x) = 0 \quad x < 0$ and $x > a$

Suppose that at a time $t = 0$ the state function describing the motion of the particle in the potential is;

$$\psi(x,0) = \frac{2b}{a}x; \quad \text{for} \quad 0 \le x \le \frac{a}{2} \quad \text{and}$$

$$\psi(x,0) = 2b\left(\frac{x}{a} - 1\right); \quad \text{for} \quad \frac{a}{2} \le x < a$$

(i) sketch the potential and the state function, [4]

(ii) determine b so that $\psi(x,0)$ is normalised. [5]

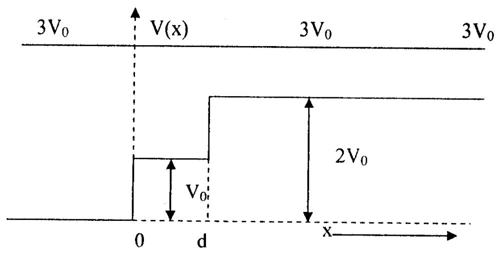
(e) Explain the spin-orbit interaction. [3]

SECTION B

2. (a) Give a physical example each for a step potential and barrier potential. [4]

(b) Draw a labelled diagram of potentials which a proton and neutron each feel when approaching a nucleus of charge +Ze. Name and express them in terms of their distance from the centre of the nucleus. [8]

(c) Consider a step potential as shown below.



(i) Calculate the transmission coefficient T and the reflection coefficient R as a function of d. [6]

(ii) What values of d give maximum and minimum transmission? [2]

3. (a) Show that $\Delta L_x \Delta L_y \geq \frac{1}{2}\hbar L_z$ where L_x, L_y and L_z are the angular momentum operators respectively. [5]

(b) If the time dependence of the function Ψ is governed by the Schrödinger equation $H\Psi = i\hbar\frac{\partial \Psi}{\partial t}$ and if the expectation value of a quantity A is given as <A>.

(i) Define $< A >$. [2]

(ii) Show that $\frac{dA}{dt} = \frac{i}{\hbar}[H, A]$ [5]

(iii) If H is the Hamiltonian for a one-dimensional Harmonic Oscillator, find the velocity of and the force on the particle executing a harmonic oscillation. [8]

4. (a) Explain why the harmonic motion is so important in physics. [4]

(b) (i) The restoring force constant μ for the vibrations of the inter atomic spacing of a typical atomic molecule is about $10^3 Jm^{-2}$. Use this value to estimate the zero – point energy of the molecular vibrations. The mass of the molecule is $4.1 \times 10^{-26} kg$. [6]

(ii) Estimate the difference in energy between the ground state and the first excited state of the vibrating molecule. From this estimate, determine the energy of the photon emitted by the vibrations when the system makes a transition between the first excited state and the ground state. [6]

(iii) Compare and state the range of the frequency of the photon emitted with the classical oscillation frequency of the system. [4]

5. A hydrogen atom is placed in a magnetic field which is very strong compared to its internal field. Its orbital and spin magnetic dipole moments precess independently about the external field, and its energy depends on the quantum numbers m_l and m_s which specify their components along the magnetic field direction.

(i) Give expressions for the total energy in terms of all the parameters involved. [3]

(ii) Evaluate the splitting of the energy levels according to the values of m_l and m_s. [5]

(iii) Draw the pattern of split levels originating from the n = 3 level enumerating the quantum numbers of each component of the pattern. [6]

(iv) Calculate the strength of the external magnetic field that would produce an energy difference between the most widely separated n = 3 levels which equals the difference between the transition of the n = 1 and n = 2 levels in the absence of an external magnetic field. [6]

- END OF EXAM -