NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 2102 – STATISTICAL MECHANICS

SUPPLEMENTARY EXAMINATION

BSC HONOURS PART II: July 2003 DURATION: 3 HOURS

ANSWER ALL PARTS OF QUESTION 1 IN SECTION A AND ANY THREE QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.

Constants

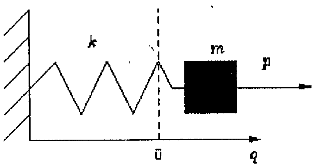
Planck's constant	h	=	6.626×10^{-34} J s
Bohr Magneton	μ_B	=	9.274×10^{-24} J T^{-1}
Boltzman constant	k	=	1.381×10^{-23} J K^{-1}

SECTION A

1 a) Define the following terms
   i) Partition function [1]
   ii) Fermi surface [1]
   iii) Microstate [1]
   iv) Bosons [1]

b) Distinguish between a microcanonical ensemble and a canonical ensemble [4]

c) Figure below shows a one-dimensional oscillator.



   i) Sketch the phase space of the harmonic oscillator [3]
   ii) Apply the *equipartition theorem* to determine the *heat capacity* of a harmonic model of a crystal of N atoms at high temperature [4]
   iii) Explain why your result in (ii) applies to high temperature only [3]

d) Suppose that the energy of an electron in a one-dimensional box of length L is given by $E = 144(\hbar^2\pi^2/2mL^2)$. How many microstates are there with energy less than or equal to E? [2]

e) Given that the entropy of an ideal gas in the semiclassical limit is given by

$$S(E,V,N) = Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\left(\frac{mE}{3\pi N\hbar^2}\right) + \frac{5}{2}N\right]$$

where the symbols have their usual meaning, show the following

i) Thermal equation of state is given by $E = \frac{3}{2}NkT$ [2]

ii) Pressure equation of state is given by $P = \frac{NkT}{V}$ [2]

f) In the vicinity of the triple point the liquid-vapor coexistence curve of liquid ammonia can be represented by $\ln P = 24.38 - 3063/T$, where the pressure is given in Pascals. The vapor pressure of solid ammonia is $\ln P = 27.92 - 3754/T$.

i) What are the temperature and pressure at the triple point? [3]

ii) What are the enthalpies of sublimation and vaporization? [4]

iii) What is the enthalpy of fusion at the triple point? [3]

g) From the third law of thermodynamics, T = 0 is a theoretically correct statement, but it is an impossibility. Why is this so? [3]

h) Calculate the partition function of an ideal gas of N =3 identical fermions in equilibrium with a heat bath at temperature T. Assume that each particle can be in one of four possible states with energies, ε_1, ε_2, ε_3 and, ε_4 and that the spin of the fermions is negligible. [3]

SECTION B

2 (a) Define the following terms

i) Fermion [2]

ii) Fermi Energy [2]

b) Given that the grand partition function for all particles in the k^{th} state can be written as

$$\mathcal{Z}_k = \sum_{n_k} \exp[-\beta n_k(\varepsilon_k - \mu)]$$

i) Define all the terms in the expression [3]

ii) Deduce the expression for the grand partition function and Landau (Grand) Potential for fermions in the k^{th} single particle state. [5]

iii) Hence or otherwise derive the expression for the mean number of fermions in the k^{th} quantum state. [3]

c) Explain why in metals the actual contribution to the heat capacity from electrons is much smaller than the prediction of the equipartition theorem and is linear in temperature as is found experimentally. [5]

a) Describe briefly the Einstein model of a solid. [3]

b) Given that the partition function of a single quantum harmonic oscillator is given by

$$Z_1 = \frac{\exp(-\beta\hbar\omega/2)}{1-\exp(-\beta\hbar\omega)}$$

For the single oscillator determine the expressions for:

i) The Helmholtz thermodynamic potential [3]

ii) Entropy [3]

iii) Mean energy [3]

iv) Hence determine the heat capacity of an Einstein solid. [3]

v) Determine the limiting behavior of the heat capacity in (iv) at low temperatures and at high temperatures and comment on its consistency with the observed behavior of solids. [5]

4 Consider a solid with N noninteracting paramagnetic atoms with spin $S = \frac{1}{2}$ and magnetic moment μ in equilibrium with a heat bath at temperature T.

a) Show that the partition function for 1 spin is given by:

$$Z_1 = \exp(-\beta\mu B) + \exp(\beta\mu B)$$ [2]

b) If the external magnetic field B = 4 T, at what temperature are 75% of the spins oriented in the direction of B? Choose $\mu = \mu_B$. [3]

c) Assume that $N = 10^{23}$, T = 1K and that B is increased quasistatically from 1 T to 10 T. What is the magnitude of the heat transfer from the heat bath? [5]

d) If the system is now thermally isolated and B is quasistatically decreased from 10 T to 1T, what is the final temperature of the system? [5]

e) Sketch the processes in (c) and (d) on a ST diagram and draw a sketch of apparatus used in practice to perform the processes (c) and (d). [5]

5 a) With the aid of a sketch of isenthalps for an arbitrary gas explain what is meant by the following terms:

i) Joule - Kelvin coefficient; [2]

ii) Inversion curve; [2]

iii) Region of heating and region of cooling [2]

b) According to Hill and Lounasmaa, the equation of the helium inversion curve is given by

$$P = -21.0 + 5.44T - 0.132T^2$$

where P is in atmospheres.

i) What is the maximum inversion temperature? [3]

ii) What point on the inversion curve has the maximum pressure? [2]

c) Sketch a graph of entropy as a function of temperature for a paramagnetic salt in an applied induction field B_0 and in zero field. [3]

d) With the aid of the sketch in (c) and appropriate thermodynamic equations for a paramagnetic salt, explain how magnetic cooling is achieved. [6]

6 a) Explain what is meant in the theory of black-body radiation by the following concepts:

i) spectral energy density $u(\omega,T)$ [2]

ii) Stefan's law [2]

iii) Wein's displacement law [2]

b) Statistical mechanics shows that at temperature T the average number of photons $\bar{n}_s$ in mode s is given by:

$$\bar{n}_s = \frac{1}{\exp(\beta\hbar\omega_s)-1},$$

where ω_s is the frequency of these photons and $\beta = 1/kT$. Use this fact to show that the spectral energy density is given by:

$$u(\omega,T) = \frac{\hbar\omega^3}{\pi^3 c^3[\exp(\beta\hbar\omega)-1]}$$

(you may use the number of photon modes per unit frequency $f(\omega) = V\omega^2/\pi^2 c^3$, where V is the volume of the system.) [6]

c) Use the above formula for $u(\omega,T)$ to show that the energy density of a black-body is proportional to T^4. [8]

END OF PAPER