

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 2102 – STATISTICAL MECHANICS

SUPPLEMENTARY EXAMINATIONS

BSc HONOURS PART II: AUGUST 2004

DURATION: 3 HOURS

ANSWER **ALL** PARTS OF QUESTION 1 IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.

1eV	=	1.602 x 10 ⁻¹⁹ J
C	=	3 x 10 ⁸ m s ⁻¹
h	=	6.63 x 10 ⁻³⁴ Js
m _p	=	1.672 x 10 ⁻²⁷ kg
amu	=	1.661 x 10 ⁻²⁷ kg

1Å	=	10 ⁻¹⁰ m
e	=	1.602 x 10 ⁻¹⁹ C
m _e	=	9.11 x 10 ⁻³¹ kg
m _n	=	1.675 x 10 ⁻²⁷ kg
a ⁰	=	0.529 Å

Stefan –

Boltzman constant, = 5.67 x 10⁻⁸ W m⁻² k⁻⁴

Wien's constant, k = 2.9 x 10⁻³ m K

Boltzman constant k = 1.381 x 10⁻²³ J/K

SECTION A

1. (a) (i) Show that for any substance if $F = F(V, T)$ then

$$C_v = -T \left(\frac{\partial^2 F}{\partial T^2} \right)_v \quad [4]$$

- (ii) If $F(V, T) = -AT \ln(V - B) - \frac{C}{V} + DT^2 + E$

Where A, B, C, D and E are constants derive the state equation for the gas. [5]

- (b) Show that the isothermal compressibility of any substance is zero at temperatures approaching zero. State the theorem or law that you have used. [6]

- (c) Show the relationship between electrostriction and the piezo-electric effect. [4]

- (d) (i) If $F(V, T) = -AT \ln(V - B) - \frac{C}{V} + DT^2 + E$

Where A, B, C, D and E are constants derive the state equation for the gas. [5]

(ii) Show that for any substance if $F = F(V, T)$ then

$$C_v = -T \left(\frac{\partial^2 F}{\partial T^2} \right)_v \quad [4]$$

(e) A paramagnetic salt obeys Curie's Law and also $C_M = A/T^2$, show that; $S = \frac{1}{2} A/T^2 - \mu_0 M^2 / 2\gamma_C + S_0$ is a constant. [5]

(f) Show that for any substance $\left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P$ and hence derive the first TdS equation. [3]

(g) The possible particle energies of a system of particles are

$$0, \varepsilon, 2\varepsilon, 3\varepsilon, \dots, n\varepsilon$$

(i) Find the expression for partition function of the system Z assuming that $g_i = 1$ for all energy levels. [4]

(ii) Compute the average energy of the particles. [2]

(iii) Find the limiting value of the average energy when $\varepsilon \ll kT$. [2]

(h) Compare the notions of statistical equilibrium and thermal equilibrium. [3]

(i) At low temperatures the rotational partition function is $Z = 1 + 3e^{-\frac{I\omega^2}{2T}}$ where I is the molecular moment of inertia. Calculate the rotational specific heat of N molecules. [6]

SECTION B

2. (a) State the principle of equipartition of energy [4]

(b) A mercury atom moves in a cubical box whose edge is 1m long. Its kinetic energy is equal to the average kinetic energy of a molecule of an ideal gas at 1000 K, if the quantum numbers n_x, n_y , and n_z are all equal to n , find an expression for n . [6]

(c) The number of particles in a system is N . the particles can only be in either of the two states with energy ε and $-\varepsilon$, but the particles do not have any translational kinetic energy.

- (i) Given that the total energy of the system is U , show that the absolute temperature T is given by

$$\frac{1}{T} = \frac{k}{2\epsilon} \ln \left(\frac{N - U/\epsilon}{N + U/\epsilon} \right) \quad [6]$$

- (ii) Verify that the absolute temperature is positive if U is negative or that it is negative if U is positive. [4]

3. (a) For a hydrostatic system, name and compare the two methods of cooling a gas adiabatically. [5]
- (b) In the joule-Kelvin effect what do you understand is the role of the inversion temperature and what are the conditions for its determination in real gases? [5]
- (c) Consider a paramagnetic material where we can neglect volume changes as a result of changes of pressure and temperature. If the material is initially isothermally magnetised by increasing the magnetism from $H = 0$ to H_1 at a temperature $T = T_1$ and then adiabatically demagnetised. Find the new temperature T_2 in terms of T_1 and H_1 given that the material obeys Curie's Law. [10]

4. (a) From Maxwell – Boltzmann distribution law it is possible to show that

$$\frac{dn}{dE} = \frac{2\pi m}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}$$

- (i) What is the meaning of the quantity dn ? [3]

- (ii) From this express for $\frac{dn}{dE}$ deduce a corresponding expression for $\frac{dn}{dv}$ where v is the velocity. [4]

- (b) Show that the number of molecules of an ideal gas that have an x – component of velocity between v_x and $v_x + dv_x$ irrespective of the values of the v_y and v_z components is

$$dn = N \left[\frac{m}{2\pi kT} \right]^{1/2} \left(e^{-m v^2 / 2kT} \right) \quad [8]$$

- (c) Compute the ratio $v_{ave} : v_{mp}$ for molecules of an ideal gas that have a velocity with only a component in the x – axis. v_{ave} is the average velocity and v_{mp} is the most probable velocity along the x – axis. [5]

5. (a) State the 3rd Law of thermodynamics otherwise also known as Nernst Theorem [2]
- (b) Show that
- (i) the expression for the entropy of an ideal gas is
$$S = C_v \ln(T) + R \ln(V) + S_o$$
where S_o is a constant. [4]
- (ii) the expression $S(V,T)$ in (i) above does not satisfy Nernst's Heat Theorem. Explain this. [6]
- (iii) the absolute zero temperature is unattainable. [8]

END OF EXAMINATION