NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

STATISTICAL MECHANICS - SPH 2102

EXAMINATION

BSc HONOURS PART II: DECEMBER 2004 *DURATION 3 HOURS*

Instructions To Candidates:

1. Answer ALL parts of question 1 in Section A.
2. Answer any **THREE** questions from Section B
3. Section A carries 40 marks and Section B carries 60 Marks.
4. Show all your steps clearly in any calculation.

SECTION A

1. (a) Define the following terms:

   (i) Microstate and Macrostate, [3]

   (ii) Partition function, [2]

   (iii) Intensive variable, [1]

   (iv) Bosons. [1]

   (b) Distinguish between a microcanonical ensemble and canonical ensemble. [4]

   (c) Given that the entropy of an ideal gas in the semiclassical limit is given by:

$$S(E,V,N) = Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\left(\frac{mE}{3\pi N\hbar^2}\right) + \frac{5}{2}N\right]$$

   where all symbols have their usual meanings, show the following

   (i) Thermal equation of state is given by $E = \frac{3}{2}NkT$ [2]

(ii) Pressure equation of state is given by $P = \frac{NkT}{V}$ [2]

(d) (i) State any difficulties you are aware of that arise with the use of Maxwell - Boltzmann statistics. [4]

(ii) State Heisenberg's principle as it relates to quantum mechanics of an electron gas. [2]

(e) Given that the Maxwell - Distribution for molecular speeds is given by the expression $f(v) = 4\pi n\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}} V^2 e^{-mv^2/2KT}$

Use the change - of - viariable techniques to convert the above expression into an energy distribution function. [4]

(f) According to Pierre Curie's theory of Paramagnetism, magnetisation is given by the expression $M = C\frac{H}{T}$, where the other symbols have their usual meanings. What could be one weakness of this formular? [2]

(g) (i) Explain why the entropy of a system is sometimes described as a measure of chaos or disorder of that system. [4]

(ii) Show that the thermodynamic probability $(\Omega) = \frac{N!}{\pi Ni!}$ can be used to derive the expression $\ln(\Omega) = N\ln N - \sum Ni\ln Ni$ with the use of Stirlings's approximation. [4]

(h) In the vicinity of the triple point, the liquid-vapour coexistence curve of liquid ammonia can be presented by InP = 24.38 N 3063/T, where the pressure is given in Pascals. The vapour pressure of solid ammonia is In P = 27.92 N 3754/T.

(i) What are the temperature and pressure at the triple point? [2]

(ii) Calculate the enthalpy of vaporisation. [2]

SECTION B

2. (a) Consider a system of N particles in a phase space of 3 cells 1,2 and 3. Suppose that the energy of a particle in cell i is given by:
$W_1 = 0$, $W_2 = W$ and $W_3 = 2W$.

(i) Write down the expressions for the partition function (z) and number of phase points in the 3 cells in terms of the characteristic temperature θ. [5]

(ii) Find an expression for the total internal energy [4]

(iii) Find an expression for the entropy [4]

(b) Describe the distribution of the particles under the following conditions.

(i) At temperatures that are very small compared to the characteristic temperature [4]

(ii) At temperatures that are very large compared to the characteristic temperature. [3]

3 (a) (i) Show that the Langevin theory for paramagnetism

$$\left[M = n\mu\left(\coth\frac{\mu B}{kT} - \frac{kT}{\mu B}\right)\right]$$

lead to Curies law in weak fields and at high temperatures.

(Given: Coth $x \approx \frac{1}{x} + \frac{x}{3}$ for a small angle) [6]

(ii) Write down the expression for the resulting curie's constant C. [2]

(iii) Explain why Langevin's theory can be said to be a more appropriate approach compared to other approaches for Paramagnetism. [3]

(b) Describe the Stern-Gerlach experiment and explain how it is used to demonstrate space quantization. [9]

4. (a) Describe briefly the Einstein model of a solid. [3]

(b) Given that the partition function of a single quantum harmonic oscillator is given by:

$$Z = \frac{\exp(-\beta\hbar\omega/2)}{1-\exp(-\beta\hbar\omega)}$$

For the single oscillator determine the expressions for :

(i) The Helmholtz thermodynamic potential [3]

(ii) Entropy [3]

(iii) Mean energy [3]

(iv) Hence determine the heat capacity of an Einstein solid. [3]

(v) Determine the limiting behaviour of the heat capacity in (iv) at low temperatures and at high temperatures and comment on its consistency with the observed behaviour of solids. [5]

5. (a) Compare and contrast classical and quantum statistics. [4]

(b) (i) By finding the thermodynamic probability of 3 phase points a, b, and c in 2 cells, i and j both under Maxwell - Boltzmann statistics and under Bose - Einstein statistics.
Show the difference between the two if the Macro state is $N_i = 2$, $N_j = 1$.
You should use diagrammatic illustrations. [6]

(ii) Use the following 2 assumptions
(1) Compartments in cell i are numbered 1, 2, 3, -------------------n.
(2) Phase points are lettered a, b, c, ------------------------N_i
to explain why the expression for thermodynamic probability is given by:

$$(\Omega) = \pi\left(\frac{n+N_i-1}{(n-1)!N_i!}\right)!$$ for Bose - Einstein statistics. [5]

(iii) Use Stirling's approximation and Lagrange's method of undetermined multipliers to show that the expression for Bose - Einstein distribution function is given by $\frac{N}{n} = \frac{1}{B\exp(\beta\varepsilon_i) - 1}$ where the all symbols have their usual meanings. [5]

6. (a) For a black body radiation, explain the following:

(i) Spectral energy density $u(E)dE$ [3]

(ii) Stefan's Law [2]

(iii) Wien's displacement law [2]

(b) (i) Statistical mechanics shows that at temperature T, the distribution function for a photon gas is $\frac{N}{n} = \frac{1}{\exp(\varepsilon i/\ kT) - 1}$

where all symbols have their usual meanings. Use this fact to show that the energy density within a frequency range dν is

$\frac{h\nu dN\nu}{V} = \frac{8\pi h\nu^3}{C^3} \bullet \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} d\nu$: Plancks law. [8]

(ii) Show how the above Planck's formular can be reduced to Rayleigh - Jeans' s classical law for black body radiation at long wavelengths. [5]

- END OF EXAMINATION -