NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

STATISTICAL MECHANICS – SPH 2102

SUPLEMENTARY EXAMINATION

BSc HONOURS PART II: JULY 2005 *DURATION 3 HOURS*

Instructions To Candidates:

1. Answer ALL parts of question 1min Section A.
2. Answer any **THREE** questions from Section B
3. Section A carries 40 marks and Section B carries 60 Marks
4. Show all your steps clearly in any calculation.

SECTION A

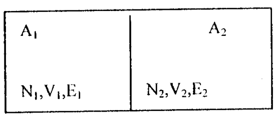
Following terms:

1 (a) Define the following terms:

(i) Microstate (2)
(ii) Partition function, (2)
(iii) Extensive variable (2)
(iv) Bosons (2)

(b) Distinguish between a canonical ensemble and a grand canonical ensemble. (4)

(c) Suppose 2 systems A_1, and A_2 are brought into thermal contact by means of a rigid impenetrable conducting wall as shown:



This system is considered to be thermodynamically isolated

(i) Show that for equilibrium, the β parameters of the two subsystems have to be equal, where $\beta = \left(\frac{\partial \ln(\Omega)}{\partial E}\right)_{N,V,E_1=\bar{E}}$ (6)

(ii) Recalling the thermodynamic relation that $\frac{\partial S}{\partial E} = \frac{1}{T}$, show that $S = k\ln(\Omega)$ where all symbols have their usual meanings. (4)

(d) For the combined systems in question *c*, show with the help of the fundamental thermodynamic relation

(dE= Tds- Pdv + μdn) that $\eta = \frac{P}{kT}$ and $\xi = \frac{-\mu}{kT}$ (4)

where μ is the chemical potential, *P* is pressure and *T*, temperature.

Given that the entropy of an ideal gas in the semiclassical limit is given by:

$$S(E,V,N) = Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\left(\frac{mE}{3\pi N\hbar^2}\right) + \frac{5}{2}N\right]$$

Where all symbols have their usual meanings, show the following

(i) Thermal equation of state is given by $E = \frac{3}{2}NkT$ (4)

(ii) Pressure equation of state is given by $P = \frac{NkT}{V}$ (4)

(e) (i) State any difficulties you are aware of that arise with the use of Maxwell-Boltzmann statistics. (4)

(ii) State Heisenberg's principle as it relates to quantum mechanics of an electron gas. (2)

SECTION B

2. (a) Given that the Maxwell – Distribution for molecular speeds is given by the expression

$$f(v) = 4\pi n\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}} V^2 e^{-mv^2/2KT}$$

Use the change- of- variable techniques to convert the above expression into an energy distribution function (5)

(b) According to Pierre Curie's theory of Para magnetism, magnetisation is given by the expression $M = C\frac{H}{T}$, where all symbols have their usual meanings.

What could be one weakness of this formula? (3)

(c) (i) Explain why the entropy of a system is sometimes described as a measure of chaos or disorder of that system (6)

(ii) Show that the thermodynamic probability $(\Omega) = \frac{N!}{\pi Ni!}$ can be used to derive the expression $\ln(\Omega) = N \ln N - \sum Ni \ln Ni$ with the use of Stirling's approximation. (6)

3. (a) Consider a system of N particles in phase space of 3 cells 1,2 and 3. Suppose that the energy of a particle in cell I is given by:
$W_1 = 0$, $W_2 = W$ and $W_3 = 2W$.

(i) Write down the expression for the partition function (z) and number of phase points in the 3 cells in terms of the characteristic temperature θ. (5)

(ii) Find an expression for the total internal energy (4)

(iii) Find an expression for the entropy (4)

(b) Describe the distribution of the particles under the following conditions.

(i) At temperatures that are very small compared to the characteristic temperature (4)

(ii) At temperatures that are very large compared to the characteristic temperature (3)

4. (a) (i) Show that the Langevin theory for paramagnetism

$$M = n\mu\left(\coth\frac{\mu B}{kT} - \frac{kT}{\mu B}\right)$$

lead to Curie's law in weak fields and at high temperatures.

(Given: $Coth(x) \approx \frac{1}{x} + \frac{x}{3}$ for small angle) (6)

(ii) Write down the expression for the resulting curie's constant C. (2)

(iii) Explain why Langevin's theory can be said to be a more appropriate approach compared to other approaches for Paramagnetism. (3)

(b) Describe the Stern- Gerlach experiment and explain how it is used to demonstrate the possession of intrinsic angular momentum and a magnetic moment by individual electrons. (9)

5. (a) For a hydrostatic system, name and compare the two methods of cooling a gas adiabatically. (5)

(b) In the Joule-Kelvin effect, what do you understand is the role of the inversion temperature and what are the conditions for its determination in real gases? (5)

(c) Consider a paramagnetic material where we can neglect volume changes as a result of changes of pressure and temperature. If the material is initially isothermally magnetised by increasing the magnetism from $H=0$ to H_1 at a temperature $T=T_1$ and the adibatically demagnetised. Find the new temperature T_2 in terms of T_1 and H_1 given that the material obeys Curie's law. (10)

6. (a) Derive the mathematical expression for the Bose-Einstein statistics which gives most probable distribution of Bosons. (10)

(b) Derive Planck's law of radiation. (10)

- END OF EXAMINATION -