# NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

## APPLIED PHYSICS DEPARTMENT

**SPH 2104 – QUANTUM MECHANICS**

**BSc HONOURS PART II: JULY 2001** **DURATION: 3 HOURS**

ANSWER **ALL** QUESTIONS IN SECTION A AND **THREE** QUESTIONS FROM SECTION B.
SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS.

## SECTION A



(a) Consider the Hamiltonian $H = \frac{p^2(t)}{2m} - eEx(t)$.

(i) What does this Hamiltonian describe and what does each term present? [6]

(ii) Deduce the equation of motion for the operators p(t) and x(t) [6]

(b) Given that $\varphi(x) = \left(\frac{\pi}{\alpha}\right) e^{-\alpha x^2/2}$ and $\int_0^\infty x^n e^{-\alpha x^2} dx = \frac{n!\sqrt{\pi}}{2^{n+1}\alpha^{(n+1+/2)}}$
Calculate:

(i) $< x^2 > and < p^2 >$ [2]

(ii) Express $\Delta x$ and $\Delta p$ in terms of the expectation values of $x$, $x^2$, $p$ and $p^2$. [2]

(iii) Estimate the minimum value of the product $\Delta x \Delta p$. [6]

(b) Evaluate the following commutators:

$$\left[\frac{d}{dx}, x^2\right],$$

$$\left[\hat{H}, x\right]$$

and $\left[\hat{H}, p_x\right]$

where $\hat{H}$ is the Hamiltonian operator in (b) [6]

(c) A neon atom in an excited state releases its excess energy by emitting a photon of wavelength 832,8 nm. If the average time between excitation of the atom and the time at which it emits a photon is $7x10^{-9}s$ estimate the uncertainty in the frequency. [5]

(d) Caculate :-

(i) the magnetic dipole moment and
(ii) the magnetic flux density at the centre of a hydrogen atom in a 2p state.

Assume that $\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{24a_0^3}$ where $a_0$ is the Bohr radius. [6]

## SECTION B

2. An electron encounters a potential step of height $1,9x10^{-19}J$. Calculate:-

(a) the probability of reflection if the electron energy is $9,6x10^{-19}J$; [6]
(b) the penetration depth if the electron energy is $1,8x10^{-19}J$; [6]
(c) sketch the wave functions in both (a) and (b) [8]

3. (a) In a non-stationary state, the expectation value of a quantity can change in time. Give an expression for the rate of change of any operator corresponding to an observable given that the Hamiltonian of the system is $\hat{H}$. [6]

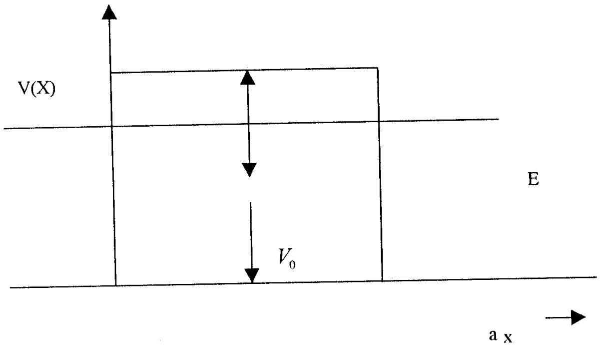
(b) The Hamiltonian of a particle is given by: $\hat{H} = \frac{\hat{P}^2}{2m} + \beta x^2$ which is independent of time.

(i) Identify the terms in the Hamiltonian and state what physical system is represented by this Hamiltonian. [4]

(ii) Find the velocity $v$ of the particle; and the force $F$ acting on the particle; [5]

4. (a) What is the tunnel effect? Give three examples and the circumstances under which the tunnel effect manifests itself. [6]

(b) Given a rectangular repulsive barrier of the following shape




Derive an equation for the probability of transmission as a function of the kinetic energy of a particle of mass $m$ and total energy $E$ incident from the left. [14]

5. (a) A particle moves under the potential $V(x) = -V_0 e^{-ax^2}$

(i) Plot V(x) [2]

(ii) Make a sketch of the wave functions when the total energy is both negative and Positive [4]

(iii) For what total energy ranges would you expect the quantization of energy? [2]

(b) (i) If the general solution of the Schrodinger equation for a harmonic oscillator is $\psi_n(x) = Q_N H_N(ax)^{-e^{a^2 x^2 / 2}}$, find the energy of the first excited state, given that

$H_n(S) = (-1)^n e^{s^2} \frac{d^n}{ds^n}\left(e^{-s^2}\right)$. Identify the quantity $Q_n$ and the function $H_n(ax)$

[6]

(ii) Calculate the zero-point energy and spacing for energy levels; in a one-dimentional harmonic oscillator with an oscillatory frequency of 400Hz. [3]

and in a three dimentional harmonic oscillator with an oscillatory frequency of 400 Hz. [3]

**END OF EXAM**