NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHSYICS DEPARTMENT

SPH 2104 - QUANTUM MECHANICS

BSc HONOURS PART II: *DECEMBER 2001* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF QUESTION 1 IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B 60 MARKS

amu	$u = 1,6604 \times 10^{-27}$kg
Planck's constant,	$h = 6.626 \times 10^{-34}$ J.s
Boltzmann constant,	$k = 1.381 \times 10^{-23}$ J/K
Avogadro's Number,	$N = 6.02 \times 10^{23}$ mol^{-1}
Electron rest mass,	$m_e = 9.11 \times 10^{-31}$ kg
Speed of light,	$c = 2.998 \times 10^{8}$ m s^{-1}
1 electron volt	$e = 1.6 \times 10^{-19}$ J
Mass of proton,	$m_p = 1.007\ 825$ u
Mass of neutron,	$m_n = 1.008\ 665$ u
1 atomic mass unit,	$u = 931.5$ MeV/c^2
Electronic Charge,	$e = 1.602 \times 10^{-19}$C
Stefan – Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ W/m^2.K^4
Wein's constant,	$k = 2.9 \times 10^{-3}$ m .K

SECTION A

1. (a) Find the angle between the angular momentum vector with quantum number l and the z – axis for all possible orientations when $l = 3$. [6]

(b) (i) What are the possible values of j for f states in a one electron atom? [4]

(ii) What are the corresponding m_j values? [2]

(iii) How many m_j states are there? [2]

(iv) How many states would there be if we instead used the labels m_l and m_s? [3]

For the ground state of the one-dimensional simple harmonic oscillator, given that the wavefunction is $\varphi_0(x) = A\exp(-\beta^2 x^2/2)$ where $\beta^2 = \frac{m\omega}{\hbar}$, $\omega^2 = \frac{k}{m}$ and $A = \sqrt{\frac{\beta}{\sqrt{\pi}}}$

(i) Evaluate $\langle x \rangle$ $\langle x^2 \rangle$ $\langle p_x \rangle$ and $\langle p^2 \rangle$ [6]

(ii) From (i) give an expression for the product $\Delta x \times \Delta p$ [3]

(d) Consider the operator $\hat{R} = -\frac{d^2}{dx^2}$ and the eigen value equation $\hat{R} = \lambda u$ where we assume that λ is a real constant. Write out the possible eigen functions and discuss the condition under which they are well behaved. [6]

(e) Show that the operator $\hat{p}_x$ is Hermitian. [3]

(f) The probability current density vector j for a particle of mass m is defined by the equation:

$$j(r,t) = -\frac{1}{2m}\left(\psi\hat{p}\psi^* - \psi^*\hat{p}\psi\right)$$

Determine the current density $j(r,t)$ for the plane wave:

$$\Psi(x,t) = A \exp \frac{i}{\hbar}\left[(2mE)^{\frac{1}{2}}x - Et\right]$$ [5]

SECTION B

2. (a) State the requirements imposed on the solutions of the Schroedinger equation. [4]

(b) Give expressions for the potential in a two – dimensional rectangular box in terms of the two co-ordinates. [4]

(c) Then find the wave functions and energy levels for a particle confined to such a two-dimensional potential box of sides a and b. [12]

3. (a) Identify and make a labelled sketch of the potentials that a proton experiences

(i) When it is among other nucleons within the nucleus. [4]

(ii) When it is approaching the nucleus from some distance r larger than the radius of the nucleus. [6]

b) Explain why fusion is more difficult to achieve than fission. Your explanation should refer to the potentials that you have identified in 3 (a) [6]

(c) Estimate the zero point energy of a proton in a nucleus given that the mass of the proton. [4]

(a) Give an expression for the z – component of the orbital magnetic dipole moment. Define all the quantities that are used and found in your derivation. [5]

(b) Describe an experiment by which the existence of the spin magnetic dipole moment of one electron atoms can be shown to have only two orientations. [5]

(c) A beam of hydrogen atoms, emitted from an oven running at a temperature of 400°K is sent through a Stern-Gerlach magnet of 0.8m length. The atoms experience a magnetic field of gradient 15.6 *tesla/m*. If the transverse deflection of a typical atom in each component of the beam, due to the force exerted on its spin magnetic dipole moment is $\pm 2.1mm$ find the value of this dipole moment. [10]

5. (a) (i) What does "Space Quantization" refer to? [3]

(ii) Describe an experiment in which space quantization manifests itself [5]

(b) A hydrogen atom is exposed to an intense magnetic field

(i) Show in a clearly labelled energy level diagram for the 3d – and – 2p states of hydrogen that in the 3d $\rightarrow$ 2p transition the number of spectral lines is three. [8]

(ii) If the magnetic field is 0.5T would the lines be observable, given that the resolution of the spectrometer is 0.10Å. [4]

6. (a) Show that for the angular momentum $L = \bar{r} \times \bar{p}$ the corresponding operator $\hat{L} = -i\hbar \bar{r} \times \bar{\nabla}$ is a Hermitian operator. [5]

) If for a physical quantity A, $\dot{A}$ is defined to mean an operator whose expectation value in any state $\Psi(r,t)$ is the time derivative of the expectation value of the operator $\hat{A}$ then show that if the Hamiltonian of the system is H then $\dot{A} = \frac{i}{\hbar}[\hat{H}, \hat{A}]$ [5]

(c) If the Hamiltonian of a system is $\hat{H} = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(x, y)$ then give expressions

(i) for the velocity components in the x, y and z directions. [5]

(ii) for the toque components in the x, y and z directions [5]

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