NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 2106 ELECTROMAGNETISM I

SUPPLEMENTARY EXAMINATION

BSC HONOURS PART II : JULY 2001 DURATION : 3 HOURS

ANSWER **ALL** QUESTIONS IN SECTION A AND ANY **THREE** QUESTIONS FROM SECTION B. SECTION A CARRIES 40 MARKS AND SECTION B CARRIES 60 MARKS

SECTION A

1. a) i) The vector force $\vec{F}$ on a conductor of length $\vec{l}$ carrying a current I in a uniform vector magnetic field $\vec{B}$ is equal to $\vec{F} = I\,\vec{l} \times \vec{B}$ (N). If $I = 10$ A, $\vec{l} = (\hat{x}2 + \hat{y}3)$ and $\vec{B} = (\hat{y}10^{-2} + \hat{z}10^{-3})$ tesla (T) find $\vec{F}$ in newtons. [Assume the direction of $\vec{l}$ to be that of the current I. [5]

ii) Find the electric flux ψ_E that passes through a surface that is bonded by the co-ordinates (3,2,0), (3,2,2), (0,2,2) and (0,2,0), when $\vec{D} = (\hat{x}y + \hat{y}x)10^{-2}\,\text{Cm}^{-2}$ [5]

b) i) From the vector magnetic potential $\vec{A}$ of the current element in (i) above, find the $\vec{B}$ by the use of $\vec{B} = \nabla \times \vec{A}$. [5]

ii) State the Uniqueness Theorem [5]

c) Use the relation $\vec{E} = -\nabla V$ to find the $\vec{E}$ field about a point charge at the origin, using the following expressions for potential difference

i) $$V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$ and [5]

ii) $$V_{a(ref)} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a}\right) + C$$ [5]

where r_a and r_b are radial distances from the origin, Q is the charge, C is a constant, V_{ab} and $V_{a(ref)}$ are the potential differences.

d) For a cylinder whose axis lies in the z-plane carrying a current of density $\vec{J}$ μAm^{-2}, evaluate $\oint_S \vec{J} \bullet \vec{ds}$ over the portion of the cylinder bounded by $r_c = 0.2$ and the surfaces at z_1 and z_2 on the cylinder. [10]

SECTION B

2\. i) Find the force of translation on an arbitrary current-carrying conductor loop in a uniform $\vec{B}$ field. [8]

ii) Using Ampere's Circuital Law, find the $\vec{H}$ field in all regions of an infinite - length coaxial cable carrying a uniform and equal current I in opposite directions in the inner and outer conductors. The axis of the cable is along the z - axis. [12]

3\. i) A parallel-plate capacitor has a plate area of 1 x 10^{-4} m^2. The plates are separated by a distance of 1 x 10^{-4} m by a dielectric material which at 1 *GHz* has the following properties: $\varepsilon_r' = 2$; $\sigma = 1\text{x}10^{-7}\, Sm^{-1}$

a) Find the equivalent circuit for this capacitor. [3]
b) Calculate the conduction current, [4]
c) the displacement current and [3]
d) the loss tangent if 1 V at 1 GHz is applied across the capacitor. [2]

ii) A circular loop is described by the equation $x^2 + y^2 = 16$ and is located in the x-y plane centred at the origin. [*See adjacent figure*]. The $\vec{B}$ filed is described by $\vec{B} = \hat{z}\, 2\sqrt{x^2 + y^2}\cos\omega t$ (T). Find the total emf induced in the loop. [8]

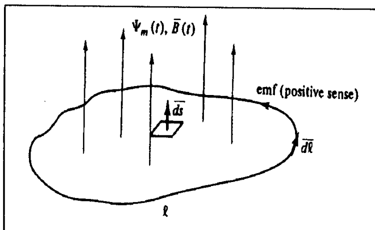

Figure 1

4\. A parallel plate capacitor with circular plates of radius R is being charged by a current I as shown in Figure 2.

a) Derive an expression for the magnetic field at radii r for the case $r \leq R$. [5]

b) Evaluate the field magnitude B for $r = \dfrac{R}{5} = 12$ mm and $dE/dt = 2.0\text{x}10^{11}\ Vm^{-1}s^{-1}$. [3]

c) Derive an expression for the induced magnetic field for the case $r \geq R$. [3]

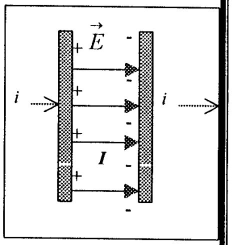

Figure 2.

d) Between the plates, what is the magnitude of $\oint \vec{B} \bullet \vec{ds}$, in terms of μ_0 and I, at a radius $r = R/5$ from their centre ? [5]

e) In terms of the maximum induced magnetic field, what is the magnitude of the magnetic field induced at $r = R/5$, inside the capacitor ? [4]

5. a) Figure 2 refers to this question.

Let the $\vec{B}$ field be directed along the positive z - axis, and let the axis of rotation be the y - axis. Sides 1 and 3 are l in length, and the sides 2 and 4 are of length w. Using the relation

$IR = V_{ab} = e.m.f. = \int \vec{u} \times \vec{B} \bullet \vec{dl}$,

show that the emf may be written as

$V_0 Sin\omega t = Bs\omega Sin\omega t$.

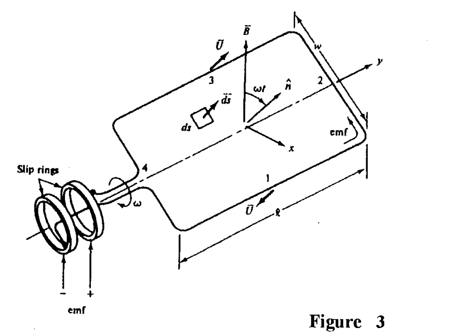

Figure 3

[12]

b) Given two finite and parallel line charges, derive the expression for the force on one of the lines

i) in terms of the field concept, charge-field-charge, [4]

ii) in terms of the action-at-a distance concept. [4]

END OF EXAMINATION