NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLIED PHYSICS DEPARTMENT

SPH 4170 - APPLIED OPTICS I

BSc HONOURS PART IV: *DECEMBER 2005* *DURATION: 3 HOURS*

ANSWER **ALL** PARTS OF QUESTION **ONE** IN SECTION **A** AND ANY **THREE** QUESTIONS FROM SECTION **B**. SECTION **A** CARRIES 40 MARKS AND SECTION **B** CARRIES 60 MARKS.

SECTION A

1. (a) i) State Fermat's principle. [1]

ii) With the aid of neat diagrams, show that both the law of reflection and Snell's law are a consequence of Fermat's principle. [6]

(b) A lens of focal length f in air is used under water. Will the effective focal length be shorter, longer or the same? Justify your answer [3]

(c) Gallium arsenide, GaAs, has a refractive index of 3.34 at a wavelength of $\lambda = 8.0\mu m$.

i) What the phase velocity of this light in GaAs? [3]

ii) For a ray incident on a flat GaAs surface in air at an angle of 30 ^{0} to the normal, what is the angle of the refracted ray inside the GaAs, with respect to normal? [3]

iii) Calculate the angle of total internal reflection for GaAs in air. [3]

(d) An object is located 100 cm from a biconvex spherical (thin) lens with radius of curvature 80 cm on each side and refractive index n = 1.5 (see figure).

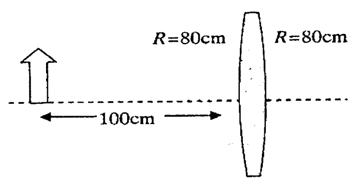


Fig. 1

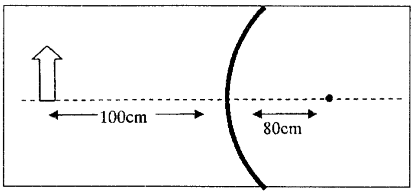
i. Calculate the focal length of the thin lens. [3]

ii. Draw the appropriate ray diagram. [3]

iii. Quantitatively describe the location, nature (real or virtual), and relative size of the image. [3]

The lens is replaced with a convex mirror with radius of curvature 80cm (see figure).

Fig. 2



[4]

iv. Draw the appropriate ray diagram.

v. Qualitatively describe the location, nature (real or virtual), and relative size of the image. [3]

(e) A complex optical system is represented by the following matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & -u \\ 0 & 1 \end{bmatrix}$$

Obtain the: **(i)** focal length, [1]

(ii) linear magnification, and [2]

(iii) angular magnification of a system. [2]

SECTION B

2. **(a)** What is the numerical aperture NA of an optical fibre? [4]

(b) (i) Explain what you understand by the normalized thickness of an optical fibre. [4]

(ii) Using a simple plane waveguide model show how these quantities are related to the number of modes that can propagate along the fibre. [6]

(c) An optical fibre has a NA of 0.15 and a core diameter of 10 μm. If the core refractive index is 1.45, estimate the wavelength that must be used to operate the fibre as a single mode fibre. [6]

(a) Ms Maguranyanga is driving along a country road at night. She sees a light approaching. Initially, it appears as a single light, but after a while the two headlights of an approaching car become discernible. Assume her eye pupil diameter is 5.0 mm, the light wavelength is 550 nm, the separation of the car headlights is 1.4 m. and the diffraction effects alone limit the resolution, so that the Rayleigh criterion can be applied.

(i) At what angular separation will her eye just resolve the two headlights? [2]

(ii) At what distance between her and the approaching car, do the lights become separately resolvable? [2]

(iii) Suppose that the general illumination around her is increased (e.g. by turning on the interior cabin light), such that her pupils shrink. Will this improve the resolvability, i.e. increase the distance at which she can resolve the headlights? Justify your answer. [2]

(b) Diffraction from a rectangular slit can be described by the equation $I = I_0 \dfrac{\sin^2 \beta}{\beta^2}$

where $\beta = \dfrac{1}{2} kb \sin\theta$ and $k = 2\pi / \lambda$.

(i) Sketch the arrangement whereby diffraction from a rectangular slit of width b onto a screen, at some (large) distance z, might be used to measure the wavelength λ of a plane wave light source. [3]

(ii) Sketch the profile of the diffracted intensity on the screen. [3]

(iii) Show that the separation distance between the first minima is given by $\Delta y \approx 2\lambda z / b$. [4]

(iv) The above method is considered to be very poor for measuring laser wavelength. Suggest two other optical methods for measuring laser wavelength and explain why they might be preferred to using single-slit diffraction. [4]

4. (a) Consider a system of two thin convex lenses of focal lengths 10 and 30 cm separated by a distance of 20 cm in air.

(i) Determine the system matrix elements and the positions of the unit planes. [4]

(ii) Assume a parallel beam of light incident from the left. Determine the image point and using the unit planes, draw the ray diagram. [4]

(b) (i) Explain the term *aberrations*. [1]

(ii) Consider a plane glass slab of thickness d made of a material of refractive index n, placed in air. Apply Snell's law to obtain an expression for the spherical aberration of the slab. What are other kinds of aberrations that the image will suffer from? [6]

(c) Explain what you understand by aplanatic points and describe the importance of aplanatic surfaces in the production of wide aperture oil immersion microscope objectives. [5]

5. (a) (i) What are the distinct vapour deposition methods used for the fabrication of optical fibres? [2]

(ii) Outline the Vapour Axial Deposition (VAD) method. [3]

(b) A 1-km optical fibre has core and cladding indices of $n_1 = 1.48$ and $n_2 = 1.46$. The core diameter is 50 μm. Find;

(i) The maximum angle that supports total internal reflection [2]

(ii) The time delay of an off-axis compared to a ray that propagates directly down the fibre, and [3]

(iii) An estimate of the modal-dispersion-limited bandwidth [1]

(c) A 2.8 km long fibre optic cable with an attenuation of 3.0 *db/km* and one splice of 0.8- *db* loss is used in a system. The source and receiver connections each exhibit 1*db* of loss. Proper system operation requires 3 μW of received optical power at the detector. Determine the required level of optical power from the light source. [3]

(d) You are tasked to design an optical network system for your small company. You must choose between Single-Mode Step-Index fibre and Multimode Step-Index fibre for your system. State and justify your choice. [6]

6. (a) The element rubidium (Rb) can emit light at two wavelengths, λ = 780 nm and 794 nm. Light from rubidium is used in a Michelson interferometer (see fig).

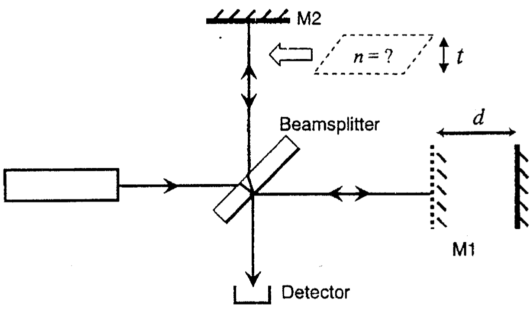


Fig. 3

The interferometer can be used to measure the refractive index of a thin transparent object of known thickness t by inserting the object into one arm while counting interference fringes. This is most easily done if only one wavelength is present. A diffraction grating can be used to separate different wavelengths of light.

(i) If mirror M2 is replaced with a reflective diffraction grating of 1200 rulings per mm, sketch the arrangement needed such that only the $\lambda = 780$ nm light is reflected back. [3]

(ii) Calculate the angle of the grating. [2]

(iii) Write an expression for the unknown refractive index in terms of its thickness t and the number of $\lambda = 780$ nm fringes counted when it is inserted into the interferometer arm. [3]

(b) Obtain the conditions of interference for the Fizeau Interferometer. [4]

(c) **(i)** Show that in the system of fringes formed in transmitted light by multiple reflection (i.e., in the system of fringes formed in a Fabry-Perot interferometer), the ratio of the intensity of the maxima to that midway between two maxima is $\left(1+R^2\right)^2/\left(1-R^2\right)$. [4]

(ii) Given a Fabry-Perot etalon with h = 1 cm and R = 0.8 (F = 80), apply the Rayleigh criterion and determine its resolving power for normal incidence in the wavelength region around $\lambda = 500$ nm. [4]

END OF PAPER

FORMULA SHEET FOR 640-237 ASTROPHYSICS AND OPTICS

Proton/electron charge	$\pm e$	$\pm 1.60 \times 10^{-19}$ C	
Proton mass	m_p	1.67×10^{-27} kg	
Permittivity of vacuum	ε_0	8.85×10^{-12} F/m (C²/N m²)	
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ H/m (Tm/A)	
Speed of light in vacuum	c	3.00×10^{8} m/s	
Planck's constant	h	6.63×10^{-34} J s	4.14×10^{-15} eV s
Gravitational constant	G	6.67×10^{-11} N m²/kg²	
Solar mass	$M_\odot$	2×10^{30} kg	
Solar luminosity	$L_\odot$	3.9×10^{26} J/s	
Solar radius	$R_\odot$	6.96×10^{8} m	
Boltzmann's constant	k	1.38×10^{-23} J/K	
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²/K⁴	
Parsec	pc	3.09×10^{16} m	
Year	yr	3.15×10^{7} s	

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i} \qquad \frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad f^2 = x_0 x_i$$

$$M_T = -\frac{s_i}{s_o} \qquad M_T = -\frac{x_i}{f} = -\frac{f}{x_o} \qquad M_L = -\frac{f^2}{x_o^2} = -M_T^2$$

$$n_i \sin\theta_i = n_t \sin\theta_t \qquad n = \frac{c}{v} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\psi(x,t) = A\sin(kx \pm \omega t) \qquad \psi(x,t) = Ae^{i(kx \pm \omega t + \varepsilon)} \qquad \psi(r,t) = \frac{A}{r}e^{i(kr \pm \omega t)}$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = c^2\varepsilon_0 \mathbf{E}\times\mathbf{B} \qquad \mathbf{E} = E_0 \sin(\mathbf{k}\bullet\mathbf{r} \pm \omega t + \varepsilon) \qquad I = \frac{1}{2}\varepsilon_0 c E_0^2$$

$$u_E = \frac{1}{2}\varepsilon_o E^2 \qquad u_B = \frac{1}{2\mu_0}B^2 \qquad u = \varepsilon_o E^2 = \frac{1}{\mu_0}B^2$$

$$\langle P\rangle = \frac{\langle S\rangle}{c} = \frac{I}{c} \qquad a(\sin\theta_i + \sin\theta_m) = m\lambda$$

$$\Delta\theta_{\min} = 1.22\frac{\lambda}{D} \qquad I = I_0 \frac{\sin^2\beta}{\beta^2},\ \beta = \tfrac{1}{2}kb\sin\theta$$

$$\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial x^2} = 0 \qquad \mathrm{FT}\{U\} = \iint U(p,q)\exp[-i2\pi(pf_x, qf_y)]\,dp\,dq$$

$$r(TE) = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \qquad r(TM) = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \qquad \mathrm{TE}: t = r + 1 \quad \mathrm{TM}: nt = r + 1$$

Part B: Modern Optics Formulae and Data Sheet

Diffraction

Fourier transforms: $F(k)=\int_{-\infty}^{+\infty} f(x)e^{-ikx}dx;\quad f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(k)e^{ikx}dk;\quad f(x)\leftrightarrow F(k)$

Fraunhofer diffraction: $U(X,Y,Z)=C\iint_A e^{-ik(x\frac{X}{Z}+y\frac{Y}{Z})}dxdy$

Fresnel diffraction: $U(X,Y,Z)=-\frac{ike^{ikZ}}{2\pi Z}\iint U(x,y)e^{i\frac{k}{2Z}[(X-x)^2+(Y-y)^2]}dxdy$

Convolution and correlation

$f\otimes g=\int_{-\infty}^{+\infty} f(x)g(X-x)dx$ If $h=f\otimes g$ then $H=FG$ If $h=fg$ then $H=\frac{1}{2\pi}F\otimes G$

$c_{fg}(\tau)=\int_{-\infty}^{\infty} f^*(t)g(t+\tau)dt$ If $f(t)\leftrightarrow F(\omega)$ and $g(t)\leftrightarrow G(\omega)$, then $c_{fg}(\tau)\leftrightarrow F^*(\omega)G(\omega)$

Imaging

Coherent imaging system: $h(x)\sim\int Q(x')e^{-i\frac{kxx'}{d_i}}dx'$

Coherent imaging: $U_i(X,Y)=\iint h(X-x,Y-y)U_g(x,y)dxdy$

Incoherent imaging: $I_i(X,Y)=\iint p(X-x,Y-y)I_g(x,y)dxdy\quad p(x,y)\propto|h(x,y)|^2$

Incoherent imaging system: $H(k_x)=\frac{\int p(x)e^{-ik_x x}dx}{\int p(x)dx}=\frac{\int Q(x)Q^*\left(x+\frac{d_ik_x}{k}\right)dx}{\int Q(x)Q^*(x)dx}$

Coherence

$g(\tau)=\frac{G(\tau)}{G(0)}=\frac{\langle U^*(t)U(t+\tau)\rangle}{\langle U^*(t)U(t)\rangle}\qquad S(\omega)\leftrightarrow G(\tau)\qquad \Delta\nu_c\approx\frac{1}{\tau_c}$

$G(\vec{r}_1,\vec{r}_2,\tau)=\langle U^*(\vec{r}_1,t)U(\vec{r}_2,t+\tau)\rangle\qquad g(\vec{r}_1,\vec{r}_2,\tau)=\frac{G(\vec{r}_1,\vec{r}_2,\tau)}{[I(\vec{r}_1)I(\vec{r}_2)]^{1/2}}$

$I=I_1+I_2+2(I_1I_2)^{1/2}|g_{12}|\cos\varphi$ where $g_{12}=\frac{\langle U_1^*U_2\rangle}{(I_1I_2)^{1/2}}$

Data

$c=3.00\times10^8$ m/s