NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
JULY 2002 EXAMINATIONS

SUBJECT: MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE
CODE: SCS1102

Instructions to candidate:

1.   Answer any four [4] questions
2.   All questions carry equal marks [25 each]

3 HOURS

QUESTION ONE

a) Is symmetric difference (+) distributive over relative compliment (-)? Use a Venn diagram as well as general proof in your answer   [5]

b) Is it always the case that for all $X,Y \subseteq \xi$, $P(X) \cap P(Y) = P(X \cap Y)$ [5]

c) Let S be a relation on $\mathbf{Z}$ such that $(x,y) \in S$ iff x-y is a multiple of 7. Prove the   following properties, of S reflexiveness, symmetry and transitivity, by direct proof.   [15]

QUESTION TWO

a)   Give a proof of: "If a+b is odd, exactly one a and b is odd"
i)   by direct proof
ii)   by contraposition
ii)   by reducio ad absurdum   [15]

b)   Given that f: $\mathbf{Z} \rightarrow \mathbf{Z}$ and f(x)=x-18, prove that f is injective.   [5]

c)   Give a direct proof of the following statement:
"*If, for any two integers, one is odd and the other is even, then their product is even*"   [5]

QUESTION THREE

By use of a proof sequence show that the following statements are tautologies.

    i) $(P' -> Q) -> (Q' -> P)$ [5]

    ii) $[L \wedge (D' -> L)] -> D$ [5]

    iii) $[(P' -> Q') \wedge (P -> S)] -> (Q -> S)$ [5]

    iv) $[(I -> H) \wedge (F \vee H') \wedge I] -> F$ [5]

    v) $[(P -> Q) \wedge Q'] -> P'$ [5]

QUESTION FOUR

Three speakers must be chosen from 20 members of a NUST technology conference, 3 are software engineers, 5 computer engineers, and 12 are theoretical computer scientists (like us!)

a)   In how many different ways can 3 successive speakers be chosen?

                                                                        [6]

b)   If the first speaker must be a theoretical computer scientist, then followed by a software engineer, and finally a computer engineer. In how many ways can the speakers be chosen?   [6]

c) How many bijective functions from X to X are there if

    i.   X={a,b,c}?   [4]

    i.   X={1,2,3,4}   [4]

    ii.   X has 113 elements   [5]

QUESTION FIVE

a)   Prove that for all X,Y,Z$\subseteq \xi$,

    X* (Y$\cap$Z) $\equiv$ (X*Y) $\cap$ (X*Z) ?   [5]

b)   Is X*(Y-W) equal (X*Y)-(X*W) for all X,Y,W$\subseteq \xi$? If so give a proof, else give a counter example   [6]

c)   Given that A={a,b}, Find:

    i.   The power set of A $P(A)$ [3]

    ii $P(A)* P(A)$ [4]

    iii   Give examples of equivalence relations on a symmetric relation $P(A)$ [4]

    iv   For your example in (iii), give the equivalence classes

                                                                        [3]

QUESTION SIX

a)   Define an ADT   [4]

b)   Define an Algorithm   [3]

c)   Give recursive algorithms for
     i.   Summing a Fibbonacci series   [6]
     ii.   Calculating the Factorial of an integer (i.e. n!)   [6]
     iii.   Tree traversals   [6]

END OF QUESTION PAPER

GOOD LUCK!