NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCS 1102

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF COMPUTER SCIENCE

MATHEMATICAL FOUNDATIONS OF COMPUTING SCIENCE

DECEMBER 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Define the following

(a) a relation from set A to set B.

(b) function from set A to set B.

(c) an injective function from set A to set B. [2,2,2]

A2. In how many ways can the symbols $a, b, c, d, e, e, e, e, e$ be arranged so that no e is adjacent to another e. [5]

A3. Let I be an index set where $\forall i \in I, A_i \subseteq \mathcal{U}$. Prove that

$$\left(\bigcup_{i\in I} A_i\right)^c = \bigcap_{i\in I} A_i^c.$$

[6]

A4. A certain "Burger Joint" advertises that a customer can have his or her hamburger with or without any or all of the following: ketchup, mustard, mayonnaise, lettuce, tomato, onion, pickle, cheese, or mushrooms. How many different kinds of hamburger orders are possible? [6]

A5. Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$.

(a) List 5 functions from A to B.

(b) How many relations from A to B are there?

(c) How many functions $f : A \mapsto B$ are there?

(d) How many functions $f : A \mapsto B$ are one-to-one?

(e) How many functions $f : A \mapsto B$ satisfy $f(1) = x$? [2,2,2,3,3]

A6. Find the value(s) of n for which $.^nP_3 = 3 .^n P_2$. [5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) If $A = \{1, 2, 3, 4, 5\}$ and there are 6720 injective functions $f : A \to B$, what is $|B|$? [4]

(b) Let $f : X \to Y$,and $\forall i \in I$, let $A_i \subseteq X$. Prove that

$$f\left(\bigcap_{i\in I} A_i\right) \subseteq \bigcap_{i\in I} f(A_i).$$

[5]

(c) (i) Define an invertible function $f : A \to B$. [3]

(ii) Prove that a function $f : A \to B$ is invertible if and only if it is a bijective function. [8]

B8. (a) Given $\mathcal{U} = \mathbb{R}$ and $I = \mathbb{R}^+$. If for each $r \in \mathbb{R}^+$, $A_r = (-1/r, 1/r)$, find

(i) $\bigcup_{r\in I} A_r$

(ii) $\bigcap_{r\in I} A_r$. [2,2]

(b) Let $\mathcal{U} = \{a, b, c, \cdots, z\}$ with $A = \{a, b, c\}, C = \{a, b, d, e\}$. If $|A \cap B| = 2$ and $(A \cap B) \subset B \subset C$, determine B. [3]

(c) Prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

[6]

(d) Given that $E_\alpha, \alpha \in I$, is a class of subsets of X, prove that

$$X \cap \left(\bigcup_{\alpha \in I} E_\alpha \right)^c = \bigcap_{\alpha \in I} (X \cap E_\alpha^c).$$

[7]

B9. (a) Prove that $(\neg P \vee Q) \wedge (P \wedge (P \wedge Q)) \Leftrightarrow (P \wedge Q)$. Write the dual of this result. [7]

(b) Negate each of the following and simplify the resulting proposition:

(i) $P \Rightarrow (\neg Q \wedge R)$

(ii) $P \vee Q \vee (\neg P \wedge \neg Q \wedge R)$. [3,4]

(c) Construct a truth table for the following proposition:

$$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R).$$

Is this statement a tautology? [6]

B10. (a) Show that if n is an integer, with $n \geq 1$, then

$$\binom{2n}{n} + \binom{2n}{n-1} = \binom{2n+2}{n+1}$$

[6]

(b) How many permutations are there of all the letters in SOCIOLOGICAL? In how many of these are the letters A and G adjacent? [6]

(c) In how many ways can 12 different books be distibuted among four children so that

(i) each child gets 3 books?

(ii) the 2 oldest children get 4 books each while the two youngest get 2 books each? [3,5]

END OF QUESTION PAPER