NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
JULY SUPPLEMENTARY EXAMINATIONS 2005

SUBJECT: MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE
CODE: **SCS1102**

INSTRUCTION TO CANDIDATES

This paper consists of **SEVEN** questions. Chose **FIVE** questions.

Time: 3 hours

QUESTION ONE

i. List all elements of the following two sets:

a) $\{x \in \mathbb{Z} \mid x^2 < 85\}$ [2]

b) $\{x \in \mathbb{N} \mid x$ is a prime number less than $50\}$ [2]

ii. Use the set builder method to describe the following two sets:

a) $\{0, 5, 10, 15, 20\}$ [2]

b) $\{2\}$ [2]

iii. What is the cardinality of each of the following sets?

a) $\{2\}$ [1]

b) $\varnothing$ [1]

c) $\{\{2\}\}$ [1]

d) $\{\varnothing\}$ [1]

e) $\{2, \{2\}\}$ [1]

f) $\{\varnothing, \{\varnothing\}\}$ [1]

g) $\{2, \{2\}, \{2, \{2\}\}\}$ [1]

h) $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$ [1]

iv. Which sets in the previous exercise (iii) the elements 2, $\{2\}$, $\varnothing$, and $\{\varnothing\}$ belong to? [4]

QUESTION TWO

i. Does every set have a subset? Why? Give an example of a set that has only one proper subset. [8]

ii. List all elements of the following sets $P(P(P(\varnothing)))$. [4]

iii. Which of the following sets is the power set of a set? Give the set if any.

a) $\varnothing$ [2]

b) $\{\varnothing, q\}$ [2]

c) $\{\varnothing, \{q\}, \{\varnothing, q\}\}$ [2]

d) $\{\varnothing, \{\{\varnothing\}\}, \{\{q, r\}\}, \{\{\varnothing\}, \{q, r\}\}\}$ [2]

QUESTION THREE

i. Why the following assignments from $\mathbb{Z}$ to $\mathbb{Q}$ are not functions:

a) $f(x)=\frac{1}{(x^2-4)}$ [4]

b) $f(x)=\pm x^2$ [4]

ii. Consider the function $f: \mathbb{R} \to \mathbb{R}$ and g: $\mathbb{R} \to \mathbb{R}$ such that $f(x)=x^2+3$ and $g(x)=x-5$ [4]

iii. Determine whether the following function from $\mathbb{R}$ to $\mathbb{R}$ is one-to-one, onto, both (i.e., a bijection), or neither:
$G(x)=x^2 - 1$ [4]

QUESTION FOUR

i. Use truth tables to prove the following logical equivalences:

a) $p \vee q \equiv (p \to \sim p) \to \sim (q \to \sim q)$ [5]

b) $p \vee q \equiv (p \to \sim q) \to \sim (p \to \sim q)$ [4]

ii. Consider the following algorithm:

1. procedure proc(n)
2. if n = 0 then
3. return(1)
4. else
5. return (proc(n-1) + proc(n-1))
6. end proc.

a) Find the output of proc(n) for any $n \geq 0$. [4]

b) Assume the complexity of this algorithm is given by the number of times the return commands are executed. Find its complexity in θ notation. [6]

c) Replace the statement in line 5 with a different statement that yields an equivalent algorithm (same output for every $n \geq 0$) of complexity $O(n)$. [1]

QUESTION FIVE

i. We have two coins. One of them is fair, i.e., the probabilities of head and tails are both equal to ½. The other one is loaded, so that the probability of getting tails after tossing it is ⅓ and the probability of head is ⅔. We choose one of the coins at random (with probability ½) and toss it.

   a) What is the probability of getting "Tails"? [5]

   b) Assume we get "tails". What is the probability that coin we just tossed is the loaded one? [5]

ii. How many strings can be formed by ordering the letters of **NORTHWESTERN** so that all **E**s appear between the 2 **N**s? [5]

iii. Find the coefficient of $x^3 y z^4$ in the expansion of $(2x + y + z)^8$ [5]

QUESTION SIX

i. Let $f : \Re \rightarrow \Re^3$ and $g : \Re^3 \rightarrow \Re$ be the following functions:

   a) f(x) = (x,x,x) [1]

   b) g(x,y,z) = x+y+z

   1. Find g o f.
   2. Find f o g. [1]
   3. Determine if g o f is one-to-one, onto or bijective and in the latter case, find its inverse. [5]
   4. Same question (3) for f o g. [5]

ii. Use induction to show that 6.7^n-2.3^n is divisible by 4, for any integer n, with $n \geq 0$. [8]

QUESTION SEVEN

Suppose that there are three boxes containing identical white, red, and blue balls. Each box contains 12 balls.

a) In how many ways can we select 12 balls? [10]
b) In how many ways can we select 12 balls such that we must have at least one ball of each color? [10]

END OF QUESTION PAPER