NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
DECEMBER EXAMINATIONS 2005

SUBJECT: MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
CODE: SCS1102

INSTRUCTION TO CANDIDATES

This paper consists of **SEVEN** questions. Chose **FIVE** questions.

Time: 3 hours

QUESTION ONE

i. List all elements of the following two sets:
   a) $\{x \in \mathbb{Z} \mid x^2 < 85\}$ [2]
   b) $\{x \in \mathbb{N} \mid x \text{ is a prime number less than } 50\}$ [2]

ii. Use the set builder method to describe the following two sets:
   a) $\{0, 5, 10, 15, 20\}$ [2]
   b) $\{2\}$ [2]

iii. Let A,B,C, be the following sets: $A=\{a, b, c\}$, $B=\{x \in \mathbb{Z} \mid 0 \leq x < 3\}$, $C=\{x \in \mathbb{Z} \mid 0 < x \leq 3\}$. Find the following sets:

   a. $B \cap C$ [2]
   b. $A \times (B \cap C)$ [2]
   c. $A \times B$ [3]
   d. $A \times C$ [3]
   e. $(A \times B) \cap (A \times C)$ [2]

QUESTION TWO

A gambler has 4 coins in her pocket. Two are double-headed, one is double-tailed, and one is normal. The coins cannot be distinguished unless one looks at them.

(a) The gambler shuts her eyes, chooses a coin at random, and tosses it. What is the probability that the lower face of the coin is heads? [2]

(b) She opens her eyes and sees that the upper face of the coin is a head. What is the probability that the lower face is a head? [5]

(c) Now, after having seen that the upper face is a head, she shuts her eyes again, picks up the same coin, and tosses it a second time. What is the probability that the lower face is a head? [6]

(d) After her second toss (as described in part (c)), she opens her eyes and sees that the upper face is a head. What is the probability that the lower face is a head? [7]

QUESTION THREE

i. Why the following assignments from $\mathbb{Z}$ to $\mathbb{Q}$ are not functions:

a) $f(x)=\frac{1}{(x^2-4)}$ [4]

b) $f(x)=\pm x^2$ [4]

ii. Let A, B, C and D be the following sets:

$A=\{(x,y)\in\mathbb{Q}^2 \mid y=x^2\}$

$B=\{(x,y)\in\mathbb{Q}^2 \mid y=x+2\}$

$C=\{(x,y)\in\mathbb{Q}^2 \mid y=2\}$

$D=\{(x,y)\in\mathbb{Q}^2 \mid x^2+y^2=20\}$

Find each of the following sets:

a) $A \cap B$

b) $A \cap C$

c) $B \cap C$

d) $A \cap D$

e) $C \cap D$

f) $A \cap D \cap (B \cup C)$ [1,1,1,1,1,3] [8]

iii. Determine whether the following function from $\mathbb{R}$ to $\mathbb{R}$ is one-to-one, onto, both (i.e., a bijection), or neither:

$G(x)=x^2-1$ [4]

QUESTION FOUR

i. Use truth tables to prove the following logical equivalences:
   a) $p \vee q \equiv (p \rightarrow \sim p) \rightarrow \sim (q \rightarrow \sim q)$
   b) $p \vee q \equiv (p \rightarrow \sim q) \rightarrow \sim (p \rightarrow \sim q)$ [5]
[5]

ii. Let R be the following relation on R:
$x \, R \, y \, . \, x - y \in \mathbb{Z}$.
Prove that R is an equivalence relation.
[10]

QUESTION FIVE

i. We have two coins. One of them is fair. i.e.. the probabilities of head and tails are both equal to ½. The other one is loaded. so that the probability of getting tails after tossing it is ⅓ and the probability of head is ⅔. We choose one of the coins at random (with probability ½) and toss it.
   a) What is the probability of getting "Tails"?
   b) Assume we get "tails". What is the probability that coin we just tossed is the loaded one? [5]
[5]

ii. How many strings can be formed by ordering the letters of **NORTHWESTERN** so that all **E**s appear between the 2 **N**s?
[5]

iii. Solve the following recurrence:
$x_n = x_{n-1} + 2x_{n-2} : x_0 = 0; x_1 = 3$.
[5]

QUESTION SIX

i. Let $f : \Re \rightarrow \Re^3$ and $g: \Re^3 \rightarrow \Re$ be the following functions:
   a) $f(x) = (x.x.x)$
   b) $g(x.y.z) = x+y+z$
      1. Find g o f.
      2. Find f o g. [1]
      3. Determine if g o f is one-to-one. onto or bijective and in the latter case, find its inverse. [1]
      4. Same question (3) for f o g. [5]
[5]