NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
MAY EXAMINATIONS 2002

SUBJECT: Advanced Mathematical Structures for Computing
CODE: SCS 2203

INSTRUCTION TO CANDIDATES

Answer all questions in Section A and any 3 in Section B
Paper contains 8 questions.

Time: 3 hours

Section A

QUESTION ONE

a) Define the following terms
- i) Sample Space
- ii) Independence (of two events A and B)
- iii) A graph G.
- iv) The expected value of the discrete random variable X with a probability distribution f(x).
- v) Random variable

[5]

QUESTION TWO

For a fair eight sided die, we may take S={1,2,3,4,5,6,7,8}, with each of the eight outcomes having probability equal 1/8. Define the events A={1,2,3,4}, B={2,4,6,8}, C={3,4,5,6}, D={4,5,6,7}, E={1,2,5,6} and F={4,8}. Which of the following events are independent?
- i) (A,B)
- ii) (A,C)
- iii) (A,D)
- iv) (A,F)
- v) (A,C,E)

[5]

QUESTION THREE

a) In a routine test for a rare disease, the probability that an individual has the disease is one in a thousand. If the in the individual has the disease the test will be positive with 99% probability. If they do not have the disease, the test will be positive with 1% probability. Find the probability that they have the disease, given that the test is positive. [5]

b) How many ways can five people be lined up to get on a bus if a certain 2 persons refuse to follow each other. [5]

QUESTION FOUR

a) i) Define and prove *the Handshaking Lemma*

ii) By definition what are conditions that must be fulfilled for two graphs G and H to be *isomorphic.*

b) Given that n: number of vertices; m: the number of edges; and d: the degree-sequence of the graph. Draw the graphs that having the following,

i) n=4 ; m=3; d (1,1,1,3)

ii) n=5 ; m=4; d (0,1,2,2,3)

c) Define a complete graph and draw the [10]

Section B

Each question is worth 20 marks

QUESTION FIVE

a) A speaks the truth 9 times out of 10 and B 7 times out of 8. A ball was drawn from a bag containing 5 white balls and 20 black balls and, A and B both state that a white was drawn. What is the chance that the ball drawn was white [8]

b) The fraction of X male runners and the fraction Y of female runners who completed marathon races is described by the joint density function

$$f(x,y) = \begin{cases} 16xy, & 0 \le x \le 1 \;,\; 0 \le y \le x \\ 0, & \text{elsewhere} \end{cases}$$

i) Find the marginal distributions *g(x)* and *h(y)* of f(x,y).

ii) Hence find the covariance, σ_{xy} of X and Y. [12]

QUESTION SIX

a) How many ways can the letters of the words DEFEATED be arranged so that the E's are separated from each other? [8]

b) Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men winds up with his own hat? [8]

c) Write Baye's formula and the conditions under which it is applicable. [4]

QUESTION SEVEN

a) State and prove Chebyshev's theorem [8]

b) Let X be the life in hours of a radio tube. Assume that X is normally distributed with mean 200 and variance σ^2. If a purchaser of such radio tubes requires that at least 90% of the tubes have lives exceeding 150 hours. What is the largest value σ can be and still have the purchaser satisfied. [8]

c) Describe the poisson distribution (probability function, mean, variance, when applicable etc). [4]

QUESTION EIGHT

[For the purposes of this question, 0 is an even number.]

a) A coin has probability p of showing heads each time it is tossed. Let Π_n be the probability that in n independent tosses it shows heads an even number of times. Show that

$$\Pi_n = p(1-\Pi_{n-1}) + (1-p)\Pi_{n-1} \qquad \text{for any } n \geq 1$$ [10]

b) Hence show by induction that

$$\Pi_n = \frac{1}{2}[1+(1-2p)^n] \qquad \text{for all } n \geq 0$$ [10]

END OF QUESTION PAPER

GOOD LUCK!