NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCS2203

FACULTY OF SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

SCS2203:ADVANCED MATHEMATICAL STRUCTURES FOR COMPUTING

JANUARY 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B.Tables of some distribution functions and some formulae for Regression Analysis are given in the APPENDIX.

SECTION A: Answer ALL questions in this section [40].

A1. The probability density function of the random variable z is given by

$$h(z) = \begin{cases} -kz & \text{for } -1 < z < 0 \\ kz & \text{for } 0 \leq z < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find

(a) the value of k; [3]

(b) the cumulative distribution function of z and sketch its graph; [5]

(c) $P(-\frac{1}{2} < z < \frac{1}{2})$. [2]

A2. Three socks are selected at random and removed in succession from a drawer containing five brown and three green socks. List the elements of the sample space, the corresponding probabilities and the corresponding values of w where w is the number of brown socks selected. Write the probability distribution and the cumulative distribution of w. Plot the graph of the cumulative distribution of w. [7]

A3. A dashboard warning light is supposed to flash red if a car's oil pressure is too low. On a certain model, the probability that the light will flash red when the oil pressure is low is 0.99. On 2% of the time the light flashes when oil pressure is not low. If there is a 10% chance that the oil pressure is low, what is the probability that a driver needs to be concerned if the oil light goes on? [8]

A4. If the probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x \leq 1 \\ \frac{1}{2} & \text{for } 1 < x \leq 2 \\ \frac{3-x}{2} & \text{for } 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of $g(x) = x^2 - 5x + 3$ [8]

A5. In a certain desert region the number of persons who become seriously ill each year from eating a certain plant is a random variable having a Poisson distribution with parameter $\lambda = 1.6$. Find the probabilities of

(a) 2 illnesses in a given year; [3]

(b) at least 7 such illnesses in 5 years. [4]

SECTION B: Answer THREE questions in this section [60].

B6. (a) An experimenter found that in a random sample of 300 women in colleges, 35% smoked cigarettes. Let p represent the population proportion of women at colleges who smoke. Can the experimenter reject the hypothesis $H_0 : p = 0.40$ in favour of the alternative hypothesis $H_1 : p \neq 0.40$ at the 5% significance level? [10]

(b) In a random sample of 400 consumers in a given city, 250 preferred Brand A cars to Brand B cars. Let p represent the population proportion of those who prefer brand A cars. Test

$$H_0 : p = 0.50$$

against

$$H_1 : p \neq 0.50$$

with $\alpha = 0.20$. [10]

B7. A detergent manufacturer (Clean Co.) is concerned about the claims by a rival firm (Bright Co.) that Bright Co. detergents result in cleaner clothes than Clean Co. detergents. Twelve dirty sheets are randomly assigned the two detergents in identical washing machines. The results are recorded on a (bright-O-meter) a machine that measures brightness with higher scores indicating brighter sheets.

Bright detergent	Clean detergent
8	7
7	5
9	8
8	10
6	6
10	6

Assuming normality of the results and that the two samples have equal variances state the relevant hypotheses. Can Clean Co. claim that Bright Co. claims are false. [20]

B8. (a) Show how to compute a $(1-\alpha)\times 100\%$ confidence interval for the population variance. [10]

(b) The standard deviation of the lifetimes of a sample of 200 electric light bulbs was computed to be 100 hours.

(i) Find the 95% confidence interval for the population variance. [5]

(ii) Find the 99% confidence interval for the population variance. [5]

B9. The mean diameter of a sample of 200 metal rings produced by a machine is 5.02mm with a standard deviation of 0.05mm. Rings of diameters within the limits from 4.96mm to 5.08mm are usable while those outside these limits are considered defective.

(a) Determine the percentage of defective rings produced by the machine. [10]

(b) What sample size n is required to achieve a 95% confidence interval similar to the interval given above? [10]

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x=1$ $p_x = 1-p$, for $x=0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

Simple Linear Regression:

$$\widehat{\sigma^2} = \frac{S_{yy} - \frac{(S_{xy})^2}{S_{xx}}}{n-2}, \quad Var(\beta_1) = \frac{\sigma^2}{S_{xx}}, \quad Var(\beta_0) = \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right)$$

END OF QUESTION PAPER