NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCS2203

FACULTY OF SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

SCS2203:ADVANCED MATHEMATICAL STRUCTURES FOR COMPUTING

JULY 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B.Tables of some distribution functions and some formulae for Regression Analysis are given in the APPENDIX.

SECTION A: Answer ALL questions in this section [40].

A1. A dashboard warning light is supposed to flash red if a car's oil pressure is too low. On a certain model, the probability that the light will flash red when the oil pressure is low is 0.99. 2% of the time the light flashes when oil pressure is not low. If there is a 10% chance that the oil pressure is low, what is the probability that a driver needs to be concerned if the oil light goes on? [10]

A2. If the weight (X) of a bag of cement is normally distributed with a mean of 40 kg and a standard deviation of 2 kg, how many bags can a delivery truck carry so that the probability of the total load exceeding 2000 kg will be 5%? [7]

A3. By definition, the median, m, of a continuous random variable X is the value of X for which $P(X > m) = P(X < m)$. Suppose that the random variable X, has probability density function given by

$$f(x) = 3x^2, \quad \text{for } 0 \leq x \leq 1.$$

(a) Find the median (m) for X. [5]

(b) Find the mean (μ) for X. [3]

(c) Comment on the difference between the two (mean and median). If X was the life span of a light bulb in years, which one of the two would you quote for advertising purposes if you are selling light bulbs. [2]

A4. A random variable X is uniformly distributed on the interval $\alpha < x < \beta$.

(a) Show that the mean of X is $\dfrac{\beta + \alpha}{2}$. [3]

(b) Show that the variance of X is $\dfrac{(\beta - \alpha)^2}{12}$. [5]

(c) Find the cumulative distribution function of X and plot its graph. [3,2]

SECTION B: Answer THREE questions in this section [60].

B5. The maximum temperature (in degrees Fahrenheit) in a certain city on any day is denoted by X with density function

$$f(x) = \begin{cases} \dfrac{2(x-65)}{75} & \text{for } 65 \leq x \leq 70, \\ \dfrac{80-x}{75} & \text{for } 70 \leq x \leq 80, \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw the graph of $f(x)$. [5]

(b) Find the probability that the maximum temperature is less than 75 degrees. [3]

(c) Find the probability that the maximum temperature is between 68 and 72 degrees. [3]

(d) Find the cumulative distribution function of X and draw its graph. [3,3]

(e) Find the mean maximum temperature of the city. [3]

B6. A quality controller is concerned about the breaking strength of a metal wire manufactured to stringent specifications. A sample of size 25 is randomly selected, and the breaking stregths recorded. The breaking strength of the wire is known to be normally distributed with variance $\sigma^2 = 9$.

(a) Find a 95% confidence interval for the mean breaking strength of the wire. [7]

(b) Interpret this confidence interval. [5]

(c) What sample size would you need to achieve a 95% confidence interval for the mean of length 4σ? [5]

(d) Why are interval estimates sometimes more desirable than point estimates? [3]

B7. (a) Briefly define the following terms:

(i) population;

(ii) sample; [1]

(iii) sample space; [1]

(iv) outcome and event. [1]

(b) A die is tossed twice and the outcomes on the two tosses recorded. [2]

(i) Write down the sample space of X with its corresponding probabilities, where X is the product of the two tosses of the die and find $E[X]$, the expected value of X. [6,2]

(ii) Write down the sample space of Y with its corresponding probabilities, where Y is the total score on the two tosses of the die and find $Var[Y]$, the variance of Y. [5,2]

B8. The aging of whisky in charred oak barrels brings about a number of chemical changes that enhance its taste and darken its colour. Shown in the table below is the change in the whisky quality (called the proof)over the time of storage.

Age(x)(in years)	Proof(y)
0	104.6
0.5	104.1
1	104.4
2	105.0
3	106.0
4	106.8
5	107.7
6	108.7
7	110.6
8	112.1

(a) Plot the graph of y against x and comment on the relationship between the two variables. [4]

(b) Obtain the parameters of the simple linear regression model

$$Y_i = \beta_0 + \beta_1 x_i + e_i$$

Where e_i is the error term. [10]

(c) Estimate the proof of a whisky which was stored for 6.73 years. [3]

(d) Is it okay to use this model to estimate the proof of a whisky which was stored for 31 years? Comment. [3]

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p,$ for $x = 1$ $p_x = 1-p,$ for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

Simple Linear Regression:

$$\widehat{\sigma^2} = \frac{S_{yy} - \frac{(S_{xy})^2}{S_{xx}}}{n-2}, \quad Var(\beta_1) = \frac{\sigma^2}{S_{xx}}, \quad Var(\beta_0) = \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)$$

END OF QUESTION PAPER