NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

COMPUTER SCIENCE DEPARTMENT

DECEMBER EXAMINATIONS 2001

SUBJECT: DISCRETE MATHEMATICS
CODE: SCS 5102

INSTRUCTION TO CANDIDATES

Answer any five (5) questions.
Each question carries 20 marks

Time: 3 hours

QUESTION ONE

a) Using predicate logic, prove the theorem

$$(\forall x)[P(x) \wedge Q(x)] \rightarrow (\forall x)P(x) \wedge (\forall x)Q(x)$$ [10]

b) Verify the correctness of the following program segment with the precondition and post condition shown:

```
{ y=0 }
 if  y<5 then
   y : = y+1

 else
    y : = 5;
{ y=1 }
```
[10]

QUESTION TWO

a) Suppose that a Prolog DataBase contains the following entries:

```
eat (bear. fish)
eat (fish. little-fish)
eat (little-fish. algae)
eat (raccoon. fish)
eat (bear. raccoon)
eat (bear. fox)
eat (fox. rabbit)
eat (rabbit. grass)
eat (bear. deer)
eat (deer. grass)
eat (wild cat. deer)

animal (bear)
animal (fish)
animal (little-fish)
```

animal (raccoon)
animal (fox)
animal (rabbit)
animal (deer)
animal (wildcat)
plant (grass)
plant (algae)

prey (X) if eat (Y,X) **and** animal (X)
Find the result of the Query in each case:

(i) **is** (eat (bear, little-fish)
(ii) **which** (X: eat(raccoon, X))
(iii) **which** (X: prey (x) and not [eat (fox,x)])
(iv) **is** (eat (fox, rabbit))
(v) **which** (Y: in-food-chain(bear, Y)) [15]

(b) Formulate a prolog rule that defines "herbivore" to add to the database of the above example in question (2a). [5]

QUESTION THREE

a) Define the terms:
i) Algorithm
ii) Recursion
iii) Tautology
iv) Partial ordering [8]

b) Write an algorithm in Pascal like Pseudocode form to determine whether $P \to Q$ is a tautology. With the use of the algorithm, determine whether a *wff* of the form

$(A \to B) \to (B^1 \to A^1)$ is a tautology or not. [6]

c) Explain the terms ***Inductive Reasoning and Deductive Reasoning.***

Provide counter examples to the following statements:

i) All animals living in the ocean are fish
ii) Input to a computer program is always given by means of data entered at the keyboard. [6]

QUESTION FOUR

a) Let A = {a,b,c,d,e}
B = {a,b,c,g,h}
C = {b,d,e,g,h,k,m,n}

Verify
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cup B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$ [10]

b) Describe what is accomplished by the Pseudocode:

if (M<N) **then**
(A) $R \leftarrow 0$
else

(A) $K \leftarrow N$

(B) **while** $(K < M)$

(i) $K \leftarrow K + N$

(C) **if** $(K = M)$ **then**

(i) $R \leftarrow 1$

(D) **else**

(i) $R \leftarrow 0$

Where N and M will represent integers. [10]

QUESTION FIVE

a) What is wrong with the following "proof" by mathematical induction?

We will prove that for any positive integer n,n is equal to 1 more than n

Assume that P(K) is true,

K = K+1

Adding 1 to both sides of this equation, we get

K+1 = K+2

Thus

P(K+1) is true. [8]

b) Let x and y be positive numbers, and prove that

X<y
If and only if
$X^2 < y^2$ [6]

c) Prove or disapprove ; "The product of any consecutive integers is even". [6]

QUESTION SIX

a) Explain the Propositional logic with a set of illustrations [5]

c) Justify each of the steps in the following proof sequence of:

$(P \rightarrow Q) \wedge [P \rightarrow (Q \rightarrow R)] \rightarrow (P \rightarrow R)$

(i) $P \rightarrow Q$

(ii) $P \rightarrow (Q \rightarrow R)$

(iii) $[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$
(iv) $(P \to Q) \to (P \to R)$
(v) $P \to R$ [15]

QUESTION SEVEN

a) Answer the following TRUE or FALSE

(i) A proof by contradiction of $P \to R$ begins by assuming both P and Q'.
(ii) The basis step of an inductive proof requires proving a property true for n=1.
(iii) Binary search is more efficient than sequential search on a sorted list of more than three elements.
(iv) The multiplication principle says that the number of elements in **A x B** equals to the number of elements in **A** times the number of elements in B.
(v) One of the De Morgan's laws states that the negation of a disjunction is the junction of the negations (of the disjuncts).
(vi) Modus ponens allow any propositional *wff* to be derived from axioms.
(vii) The domain of an interpretation consists of the values for which the predicate *wff* defined on that interpretation is true.
(viii) If an assertion after an assignment statement is y>4, then the precondition must be $y \geq 4$. [8]

b) Show that the solution to the recurrence relation:

$S(1) = 1$

$S(n) = 2S(\frac{n}{2}) + 1$ for $n \geq 2, n = 2^m$

is (2n-1) [Hint: use $2^{\log n} = n$] [8]

c) Use formal logic notation to define $A \subset B$ [4]

QUESTION EIGHT

a) Show that the following argument is valid "All rock music is loud music. Some rock music exists, therefore some loud music exists". [5]

b) Give a direct proof of the theorem "if an integer is divisible by 6, then twice that integer is divisible by 4". Show each step is going from hypothesis to conclusion. [5]

c) Prove De Moivre's theorem:

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

for all $n \geq 1$. [5]

[Hint: use the trigonometric addition formulae given below]

$Cos(\alpha + \beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta$

$Sin(\alpha + \beta) = Sin\alpha Cos\beta + Cos\alpha Sin\beta$

d) Which of the following are true for all sets A and B

(i) if $A \subseteq B$ *and* $B \subseteq A$, *then* $A = B$

(ii) $\{\phi\} = \phi$

(iii) $\phi \in A$

(iv) $\{\phi\} = \{10\}$

(v) $\phi \in \{\phi\}$ [5]

END OF QUESTION PAPER

GOOD LUCK!