NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCS5103

DEPARTMENT OFCOMPUTER SCIENCE

DISCRETE MATHEMATICS

December 2002

Time : 3 hours

Candidates should attempt **FOUR** questions, with at least **one** question from each Section.

SECTION A: LOGIC

A1. (a) Let A and B be statements and T a tautology. Give the truth table for each of the following compound statements:

(i) $\neg B \Leftrightarrow (A \vee \neg T)$ [3]

(ii) $T \rightarrow (A \wedge B)$ [3]

(b) (i) Establish the following tautologies:

(a) $A \wedge A \leftrightarrow A$ [3]

(b) $\neg(A \wedge B) \leftrightarrow (\neg A) \vee (\neg B)$ [3]

(c) $\neg(A \vee B) \leftrightarrow (\neg A) \wedge (\neg B)$ [3]

(ii) Define $A|B$ to mean $\neg(A \wedge B)$ *(this is the Sheffer stroke function)*. Express $\neg A, A \vee B, A \wedge B, A \rightarrow B$ and $A \leftrightarrow B$ in terms of $|$. [5]

(iii) $P(A, B, C)$ is defined to be true if precisely one of A, B, C is true. Express P in terms of $\vee, \wedge, \neg$, and hence in terms of $|$. [5]

[25]

A2. (a) Prove that for any sets A, B if $A \subseteq B$ then $P(A) \subseteq P(B)$. [5]

(b) Let A, B, X, Y be sets such that $B = X \cup Y$. Show that

$$A \times B = (A \times X) \cup (A \times Y)$$

[5]

(c) Let X be a nonempty set and P(X) its power set. Define a relation $\sim$ on $P(X)$ by:

$$A \sim B \leftrightarrow A \subseteq B.$$

Investigate whether $\sim$ is reflexive, symmetric, antisymmetric or transitive. [2,2,2,2]

(d) Give the truth table for the following formula

$$((p \wedge \neg q) \vee r) \rightarrow ((r \vee p) \leftrightarrow (r \vee q))$$

[7]

[25]

SECTION B: ALGORITHMS

B3. (a) Define an algorithm. [2]

(b) Write a recursive algorithm to generate the next term of a Fibonacci series. [3]

(c) Give a recursive algorithm for a binary search. [5]

(d) Explore the complexity (using the Big-O Notation) of the binary search. Explain your answer. [5]

(e) Give a recursive algorithm for *inorder, preorder and postorder* tree traversals. [3]

(f) Draw a binary tree that produces the following output under the mentioned traversals method:

(i) inorder: B,A,D,C,E

(ii) preorder: A,B,C,D,E

(iii) postorder: D,E,C,B,A

[7]

B4. (a) Define the following:

(i) equivalence class

(ii) equivalence relation

(iii) weak partial order relation

[6]

(b) By use of direct proof, show that the set of all parallel lines are an equivalence relation. [4]

(c) Give the bubble sort algorithm:

```
for (i = 0 to n)
{ for (j = 0 to n)
if (array[i] < array[j])
swap (array[i], array[j]);
}
```

With explanation analyse the complexity of this code segment. [8]

(d) Verify the following piece of code :

$$for\ (i = 0\ to\ n)$$
$$i = i + 1$$

[7]

SECTION C: GRAPH THEORY

C5. (a) Define the following terms:

(i) graph G

(ii) Hamiltonian Cycle

(iii) Null Graph

(iv) isomorphism of G and H

[4]

(b) Prove the Handshaking Lemma. [2]

(c) Let us define a relation R on graphs by $G_1 R G_2$ if G_1 isomorphic to G_2. Is R symmetric? [6]

(d) Prove by induction that the number of edges in K_n is $n(n-1)/2$. [6]

(e) Suppose we have a network of n computers each connected to others. One of the computers on this network has a virus that will only pass to an uninfected machine or the one on which started (in which case it will stop). Is there a way for the virus to pass through each of the machine and then stop on the machine it started on? [7]

C6. (a) Define the following: *Eulerian Graph, Planner Graph, Complete bipartite Graph, Hand Shaking Lemma.* [4]

(b) Draw two (non isomorphic) simple graphs on four vertices. [1]

(c) Is it possible to have a group of 9 people such that each person is acquainted with exactly five of the other people? Either draw a graph or give reasons for your response. [4]

(d) Suppose you run a day care for an office building and there are seven children A,B,C,D,E,F,G. You need to assign a locker where each child's parents can put the child's food. The children come and leave so that they are not all there at the same time. You have one hour time slots, starting 7 am to 12 noon. A * means a child is present at the time. What is the minimum number of lockers necessary? Show how you would assign lockers? See the following table for information.

	A	B	C	D	E	F	G
7:00	*	-	-	*	*	-	-
8:00	*	*	*	-	-	-	-
9:00	*	-	*	*	-	*	-
10:00	*	-	*	-	-	*	*
11:00	*	-	-	-	-	*	*
12:00	*	-	-	-	*	-	-

[12]

(e) There is a theorem that states: *"If a simple graph has a Euler circuit then all the vertices have even degree"*. Consider the statement *If all the vertices of a simple graph have even degree the graph has a Euler circuit"*. That statement is the converse of the Theorem. Is it a true statement? **Hint:**find a simple graph with all vertices of even degree but no Euler circuit. [4]

END OF QUESTION PAPER