NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCS 5102

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF COMPUTER SCIENCE

SCS 5102: DISCRETE MATHEMATICS INTRODUCTION

AUGUST 2004 SUPPLEMENTARY

Time : 3 hours

Candidates should attempt **ALL** questions

A1. (a) Let A be a set and let $\{B_i\}_{i\in\mathbb{N}}$ be a family of sets. Show that
$A - \bigcap_{i\in\mathbb{N}} B_i = \bigcup_{i\in\mathbb{N}}(A - B_i)$ [7]

(b) Give the truth table for the following formula
$((P \wedge \sim Q) \vee R) \Rightarrow ((R \vee P) \Leftrightarrow (R \vee Q))$ [6]

A2. (a) Define an order relation. [3]

(b) On the set $\mathbb{R}$ of real numbers, define a relation $\sim$ by $x \sim y \Leftrightarrow x^2 = y^2$. Determine whether or not $\sim$ is an equivalence relation. [7]

A3. (a) Prove the following identities on the non-empty sets A, B and C

(i) $(A - B) - C = A - (B \cup C)$ [5]

(ii) $(A - B) \cap C = (A \cap C) - (B \cap C)$ [5]

(b) Prove by induction the following Bernoulli's inequality:
$(1 + \alpha)^n \geq 1 + n\alpha$ for all $n \in \mathbb{N}$ and $\alpha > -1$ [7]

A4. Let M = (S, J, O, v, w) be a finite state machine where $S = \{s_0, s_1, s_2, s_3\}$; J = (a,b,c) and O = (0,1) and v and w are given in the table below

	v	w
	a b b	a b c
s_0	s_0 s_3 s_2	0 1 1
s_1	s_1 s_1 s_3	0 0 1
s_2	s_1 s_1 s_3	1 1 0
s_3	s_2 s_3 s_0	1 0 1

(a) Starting at s_0, what is the output for the input string **abbccc**? [5]

(b) Draw a state diagram for this finite state machine. [7]

(c) In which state, if any, should we start so that the input string **abccb** produces an output **10010**? [6]

A5. (a) Let G = (V,E) be an undirected graph. Define a relation $\mathcal{R}$ on V by $a\mathcal{R}b$ if a = b or there is a path in G from a to b. Prove that $\mathcal{R}$ is an equivalence relation. [7]

Describe the partition of V induced by $\mathcal{R}$. [4]

(b) If G = (V,E) is an undirected graph with $|V| = v$ and $|E| = e$ and no loops. Prove that $2e \leq v^2 - v$. State the corresponding inequality for the case when G is directed. [7,3]

A6. Let G = (V,E), H = (V',E') be undirected graphs with $f : V \rightarrow V'$ establishing an isomorphism between the graphs

(a) Prove that $f^{-} : V' \rightarrow V$ is also an isomorphism for G and H. [6]

(b) If $a \in V$, prove that deg(a) (in G) = deg(f(a)) (in H). [7]

(c) Prove that vertex a is isolated (pendant) in G if and only if f(a) is isolated (pendant) in H. [8]

END OF QUESTION PAPER