NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
JULY SUPPLEMENTARTY EXAMINATIONS 2005

SUBJECT: DISCRETE MATHEMATICS
CODE: SCS5102

INSTRUCTION TO CANDIDATES

Answer any four questions. Paper contains five questions.

Time: 3 hours

QUESTION ONE

a) Define a relation ***R*** on **N** $\times$ **N** by $(a, b)\ R\ (c, d)$ if and only if $a + b = c + d$. Prove that R is an equivalence relation on **N** $\times$ **N**. [10]

b) Let S denote the set of equivalence classes of R. Show that there is a one-to-one and onto function from S to N. [8]

c) Use mathematical induction to prove that the statement is true for every positive integer n.

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$

[7]

QUESTION TWO

a) Prove the following assertions:

i) The sentence $(\overline{P \leftrightarrow q}) \leftrightarrow ((p \vee q) \wedge (\overline{p \wedge q}))$ is a tautology.[7]

ii) Without using the truth tables show that:

$$(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r)) \quad [8]$$

b) Solve the recurrence relation subject to the basis step by using the expand guess, and verify approach.

F (n)=1

F (n)=nF (n -1) [10]

QUESTION THREE

a) For each of the following sentences, write down the sentence in logical notation, negate the sentence, and say whether the sentence or its negation is true:

i. There is no greatest natural number [2]

ii. Every integer is a sum of the squares of two integers [2]

iii. Every even number is a sum of two odd numbers. [2]

iv. Every odd number is a sum of two even numbers. [2]

v. Given an integer there is a lager integer [2]

b) Suppose that $I = O = \{0,1\}$

i) Design a finite state machine that will recognize the pattern 10010. [8]

ii) Design a finite state machine that will recognize the pattern 10010, but only when the last 0 in the sequence pattern occurs at a position that is a multiple of 5. [7]

QUESTION FOUR

2. Using the analogues in logic demonstrate the following deductions in sets:

i) $P \cup (Q \cap R = ((P \cup Q) \cap (P \cup R))$ [9]

ii) $P \cap (Q \cup R = ((P \cap Q) \cup (P \cap R))$ [9]

iii) $\overline{P \cup Q} = \overline{P} \cap \overline{Q}$ [7]

QUESTION FIVE

a) Let A = {a, b, c, d, n, p}
B = {a, b, e, g, h}
C = {b, d, e, g, h, m, n}

Verify:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cup B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

[10]

a) Describe the following search Algorithms. Exemplify where possible.

i) Depth-First Search [5]

ii) Breadth-First Search [5]

iii) Shortest Path Problem [5]

END OF QUESTION PAPER