NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCE
COMPUTER SCIENCE DEPARTMENT
DECEMBER EXAMINATIONS 2005

SUBJECT: DISCRETE MATHEMATICS
CODE: SCS5102

INSTRUCTION TO CANDIDATES

Answer any four questions

Time: 3 hours

QUESTION ONE

a) Suppose that **P, Q, R** are sets with respect to the universe **U**. Prove the following

i) $P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$ [10]

ii) $\overline{(P \cap Q)} = \overline{P} \cup \overline{Q}$ [7]

b) Define the following Terms:

i) Inductive Proof, exemplify [5]

ii) Predicate [3]

QUESTION TWO

a) Consider the set $P = \{1, 2, 3, 4, 6, 9\}$. Define a relation R on P by writing $(x, y) \in R$ if and only if $x - y$ is a multiple of 2.

i) Describe R as a subset of $\boldsymbol{P} \times \boldsymbol{P}$. [3]

ii) Show that R is an equivalence relation on P. [6]

iii) What are the equivalence classes of R? [6]

b) Using a the predicate logic prove the theorem:

$$(\forall x)[P(x) \wedge Q(x)] \rightarrow (\forall x)P(x) \wedge (\forall x)Q(x)$$ [10]

QUESTION THREE

a) Confirm the following logical equivalencies

i) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ [4]

ii) $\overline{(p \vee q)} \Leftrightarrow \bar{p} \wedge \bar{q}$ [3]

iii) $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$ [4]

b) Check whether or not the logical implications hold for the following pairs of statements, say if the pair is logically equivalent. Consider P (x) as "*x is even*" and Q (x) as "*x is odd*" and also consider the Universe of all integers.

i) $\forall x.P(x) \vee Q(x)$ and $(\forall x.P(x)) \vee (\forall x.Q(x))$ [7]

ii) $\forall x.P(x) \rightarrow Q(x)$ and $(\forall x.P(x)) \rightarrow (\forall x.Q(x))$ [7]

QUESTION FOUR

a) Let A = {a, b, c, d}

B = {a, b, e, g, h}

C = {b, d, e, g, h, m, n}

Verify:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cup B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

[10]

b) Justify each of the steps in the following proof sequence of:

$(P \rightarrow Q) \wedge [P \rightarrow (Q \rightarrow R)] \rightarrow (P \rightarrow R)$

i) $P \rightarrow Q$

ii) $P \rightarrow (Q \rightarrow R)$

iii) $[P \rightarrow (Q \rightarrow R)] \rightarrow [(P \rightarrow Q) \rightarrow (P \rightarrow R)]$

iv) $(P \rightarrow Q) \rightarrow (P \rightarrow R)$

v) $P \rightarrow R$ [15]

QUESTION FIVE

a) Use mathematical induction to prove that the statements are true for every positive integer **n**.

i) $1+2+3+\ldots+n=\frac{n(n+1)}{2}$ [8]

ii) $1^2-2^2+3^2-4^2+\ldots.+(-1)^{n+1}n^2=\frac{(-1)^{n+1}n(n+1)}{2}$ [8]

b) Let x, y, m, n, a, b, c, d $\in$ Z satisfy m = ax + by and n = cx + dy with ad - bc = ±1. Prove that (m, n) = (x, y). [9]

QUESTION SIX

a) Suppose that T = O = {0, 1, 2, 3, 4, 5}.

i) Design a Finite state machine, which inserts the digit 0 at the beginning of any string beginning with 0,2 or 4, and which inserts the digit 1 at the beginning of any string beginning with 1,3 or 5. Describe your result in the form of a transition table. [10]

ii) Design a finite state machine, which replaces the first digit of any input string beginning with 0,2 or 4 by the digit 3. Describe your result in the form of a transition table. [10]

b) Let X and Y be positive numbers, and prove that $X<Y$ if and only if $X^2<Y^2$. [5]

END OF QUESTION PAPER