NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF APPLIED SCIENCES
COMPUTER SCIENCE DEPARTMENT
JANUARY EXAMINATIONS 2013

SUBJECT: DISCRETE MATHEMATICS
CODE: SCS5102
INSTRUCTION TO CANDIDATES
This paper consists of five questions. Answer any FOUR questions.
Each question carries 25 marks **Time: 3 hours**

Question One

a) Given that M = {a: a ∈Z and 1< a <6} and T = {a: a∈ Z and 1<a<3} where Z is a set of integers with a representing some elements of Z.
List the elements of:
   i) M∪T [3]
   ii) T∩M [1]
   iii) MxT [1]

b) Let set A={x: x ∈ R: $x^2 + 8x \le -15$} and set B = {x: x∈ R}, where R is a set of Real Numbers and x represents some elements of set R. Prove that A ⊂ B. [4]

c) Using set identities and De Morgan's laws prove that:
$(P \cup Q)' \cap (P \cap Q)' = (P \cup Q)'$ [6]

d)

e) Define an equivalence relation on Z given that x is equivalent to y if $x - y$ is divisible by 7. [5]

f) Use mathematical induction to prove that the statement is true for every positive integer n. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ [5]

Question Two

Below are two graphs, G1 has vertices labelled as a1, a2, a3, a4 and G2 has vertices labelled as g1, g2, g3 and g4.

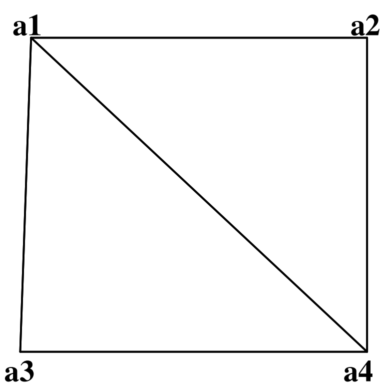

Figure 1: G1

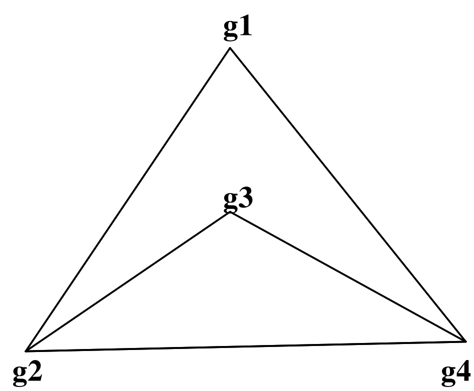

Figure 2 : G2

a) Determine the relationship between G1 and G2 in terms of their valencies. [4]
b) Construct the isomorphism for G1 and G2. [6]
c) Construct the adjacent matrices for G1 and G2. [4]
d) Permute the rows and columns for the adjacent matrices of G1 and G2. [6]

e) Comment on the relationship between G1 and G2. [5]

Question Three

a) Investigate whether the following are Tautologies or not:

i) $\overline{(p \vee q)} \leftrightarrow (\overline{p} \wedge \overline{q})$. [6]

ii) $(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$ [7]

b) Prove the following deductions in sets:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [8]

b) i) Given that f(x) = x^2. Show that f(x) is not injective under real numbers. [2]

ii) Given that $f(x) = x^2 - 3$ and $g(x) = x + 1$. Find g($f(x)$). [2]

Question four

a) Explain the following as applied to graph theory:
   i) Euler circuit [2]
   ii) Euler trail [2]
   iii) Euler path [2]
   iv) Hamiltonian cycle [2]

b) Figure 3 below is a graph G3 whose vertices are labeled as a, b, c, d and e

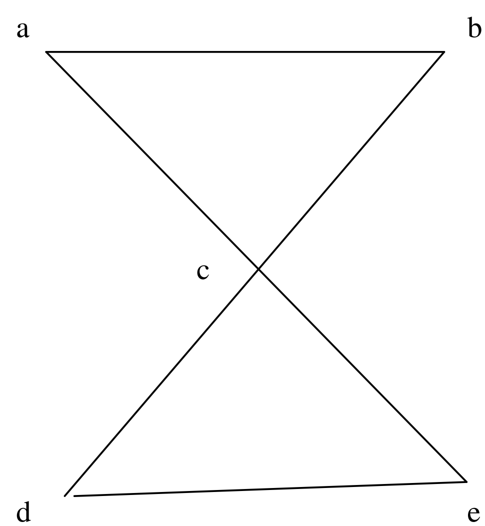

Figure 3: **G3**

Analyse the graph in terms of :

i) Euler circuit [4]
ii) Euler trail [4]
iii) Euler path [4]
iv) Hamiltonian cycle [5]

Question five

a) What do you understand by:
   i) Strong mathematical induction [2]
   ii) Weak mathematical induction [2]

b) Prove by induction for $n \geq 1$ that:
   $1x1! +2x2!+3x3!+..+nxn! = (n+1)! - 1$ [7]

c) Using Euclidean Algorithm, find the greatest common divisor of (210,858). [4]

d) Suppose that T=O={0,1,2,3,4,5}.
   Design a finite state machine which replaces the first digit of any input string beginning with 0, 2 or 4 by the digit 3. Describe your result in the form of a transition table. [10]