National University of Science and Technology
Faculty of Applied Science

Department of Computer Science

December 2002 Examination

**Digital Signals Processing SCS 6105**

Maximum time allowed: **3 Hours**
Answer any **four** questions. Each question carries **25** marks.

1. a) Show that a discrete-time signal is periodic if and only if its frequency is a rational number. **[5]**
   b) Consider the analog signal

$$x_a(t) = 3\cos 60\pi t.$$

   (i) Sketch the signal $x_a(t)$ for $0 \le t \le 10$. **[3]**

   (ii) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/sec. Determine the frequency of the discrete-time signal $x(n) = x(nT)$. Show that this signal sampled at the above rate is periodic. **[7]**

   (iii) Compute the sample values in one period of $x(n)$. Demonstrate graphically, on the same diagram, the values of $x(n)$ and $x_a(t)$. **[4]**

   (iv) Is it possible to find a sampling rate such that $x(n)$ is maximum? What is the minimum value of $F_s$ that will produce this result. **[6]**

2. a) Let a discrete-time signal be given by

$$x(n) = \begin{cases} 3 + \dfrac{3}{n} & \text{for } -4 \le n \le -1 \\ 0, & \text{elsewhere} \end{cases}.$$

   Sketch the signals that result if we :

   (i) First fold $x(n)$ and then advance the resulting signal by three samples. **[3]**

   (ii) First delay the signal $x(n)$ by three samples and then fold the resulting signal. **[3]**

   b)The input-output pair

$$x_0(n) = \{0,0,3\} \overset{T}{\leftrightarrow} y_0(n) = (0,1,0.2)$$

   was observed during the operation of a time-invariant system. Say whether the system is linear or not. What is the impulse response of the system? **[7]**

   c) Determine and sketch the convolution $y(n) = x(n) * h(n)$ of the signals

$$x(n) = \left(\frac{1}{2}\right)^n u(n) \text{ and } h(n) = \left(\frac{1}{3}\right)^n u(n). \quad \textbf{[7]}$$

   d) The input-output equation for a relaxed system is given by

$$y(n) = ny(n-1) + x(n) \text{ for } n \ge 0.$$

   Determine whether the system is linear time invariant and BIBO stable. **[5]**

3. a) Determine the zero-input response of the system described by the second order difference equation
$y(n) - 4y(n-1) - 5y(n-2) = 0$. [7]

b) Consider the system described by the difference equation

$$y(n) = \frac{5}{6}(n-1) - \frac{1}{6}y(n-2) + x(n).$$

If the forcing function

$$x(n) = 2^n u(n)$$

is applied to the system what is the particular solution of the difference equation. [12]

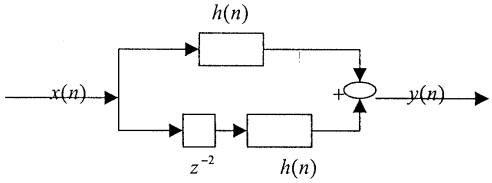
c) Determine the impulse response and the unit step response of the system described by the difference equation

$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$. [6]

4. a) A system with impulse response $h(n) = a^n u(n)$, $-1 < a < 1$ is described in a diagram below.

Find the response $y(n)$ of the system to the excitation

$$x(n) = u(n-5) + u(n+10).$$




[7]

b) Calculate the zero-state response of the system described by the difference equation

$$y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$$

to the input

$$x(n) = \{1,2,3,4,2,1\}$$

by solving the difference equation recursively. [10]

c) Determine the autocorrelation sequences of the signal s

$$x(n) = \{1,2,1,1\} \text{ and } y(n) = \{1,1,2,1\}$$

What conclusion do you draw? [8]

5. a) Determine and sketch the magnitude and phase spectra of the periodic signal

$$x(n) = \cos\frac{2\pi}{3}n \sin\frac{2\pi}{5}n.$$ [7]

b) Determine the periodic signal $x(n)$, with fundamental period $N = 8$, if its Fourier coefficient is given by

$$c_k = \begin{cases} \sin\dfrac{k\pi}{3}, \text{ for } 0 \le k \le 6 \\ 0, \text{ for } k = 7 \end{cases}.$$ [6]

c) Evaluate the Fourier transform of the signal

$$x(n) = \left(\frac{1}{4}\right)^n u(n+4).$$ [5]

d) Determine the signal having the Fourier transform:

$$X(\omega) = \begin{cases} 1 & \text{for } \omega_0 - \delta\omega/2 \le |\omega| \le \omega_0 + \delta\omega/2 \\ 0 & \text{for all other values of } |\omega| \end{cases}$$ [7]

6. a) Calculate the magnitude and the phase response of the multipath channel

$$y(n) = x(n) + x(n-M).$$

At what frequencies is $H(\omega) = 0$?. [8]

b) Let the lowpass filter be described by the system equation

$$y(n) = ay(n-1) + bx(n), \text{ for } 0 < a < 1.$$

Determine

i) $b$ so that $|H(0)| = 1$.

ii) the 3-dB bandwidth $\omega_3$ for the normalized filter in part i).

iii) How does the choice of the parameter affect $\omega_3$.

iv) Do part i) and ii) for the highpass filter obtained by choosing $-1 < a < 0$. [12]

c) Determine if the FIR system is minimum phase:

$h(n) = \{5,-3,-4,0,2,1\}$. [3]

d) Explain why a system with impulse response

$$h(n) = a_0\delta(n) + a_1\delta(n-D) + a_2\delta(n-2D)$$

generates echoes spaced $D$ samples apart. [2]