NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

COMPUTER SCIENCE DEPARTMENT

DECEMBER EXAMINATIONS 2004

SUBJECT: DIGITAL SIGNALS PROCESSING

CODE: SCS6105

INSTRUCTION TO CANDIDATES

- 1. Answer any **five** questions only.
- 2. Each question carries equal marks.
- 3. Show all your steps clearly in any calculation.
- 4. Start the answers for each question on a fresh page

Time: 3 hours

QUESTION ONE

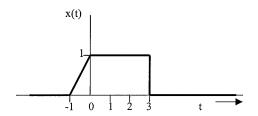


Fig 1

Fig 1 shows a continuous signal x(t) .Sketch and label the signal found after processing x(t)

- (i) x(t-1)
- (ii) x(3-t)
- (iii) x(2t)
- (iv) x(t+2)u(t)

(12)

- (b) Give a brief description of independent variables found in signals. .Give examples of signals discussed. (5)
- (c) State the characteristics of signals that are needed in order to choose the method of extraction of information in a signal. (3)

QUESTION TWO

(a) For the linear time invariant in fig 2 give the expression for the impulse response.

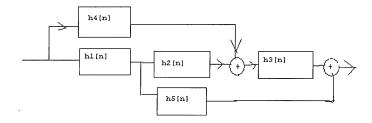


Fig 2 (6)

(b) Determine the Fourier transform of

$$x(t) = e^{-\alpha|t|} \text{ where } \alpha > 0$$
 (8)

(c) Sketch the graphical representation of the signal and its transform in (b) above . (6)

QUESTION THREE

- (a) Draw the block diagram that would represent a simplified hardware architecture for a special purpose digital signal processor. Give an explanation of the diagram. (14)
- (b) State at least four characteristics that would be included in the specification of a digital filter. (4)
- (c) Give the major difference between the finite impulse response digital filter and the infinite impulse response filter. (2)

QUESTION FOUR

- (a) Draw a timing diagram to illustrate the concept of pipelining (2)
- (b) State three special instructions optimised for digital signal processing. (3)
- (c) Give at least four instructions performed in one cycle by the MACD .(4)
- (d) Find the z transform of $x[n]=[3(2^n)-4(3^n)]U[n]$ (5)
- (e) Show a block diagram that will produce the sequence output y[n] from x[n].

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + a_1 y[n-1] + a_2 y[n-2].$$
 (6)

QUESTION FIVE

Explain the complete characterisation of a continuous time linear time invariant system in terms of a unit impulse response . (20)

QUESTION SIX

- (a) Give at least seven advantages of digital filters that make them favoured for use in in DSP. (7)
- (b) Sketch the basic structure of a lattice digital filter. (5)
- (c) Draw the structure to represent the equation below.

Write the difference equation for the structure.

(8)

$$H(z) = \sum_{k=0}^{11} h(k)z^{-k}$$

QUESTION SEVEN

(a) Find the inverse z-transform of the following:

$$X(z) = \frac{z^{-1}}{1 - 0.25z^{-1} - 0.375z^{-2}}$$
(10)

(b) Sketch the pole and zero diagram of the digital filter given by the transfer function.

$$H(z) = \frac{1 - z^{-1} - 2z^{-2}}{1 - 1.75z^{-1} + 1.25z^{-2} - 0.375z^{-3}}$$
(10)

QUESTION EIGHT

- (a) Give the methods used in calculating filter coefficients in the study of DSP . State and justify which method you should use in each of the following application $\frac{1}{2}$
- (1) Phase (delay) equalization for a digital communication system,
- (2) Simulation of analogue systems.
- (3) Image processing
- (4) High quality processing of audio systems
- (5) A high throughput noise reduction system requiring a sharp magnitude frequency response filter.
- (6) A real time biomedical signal processing with minimal distortion. (12)
- (b) Calculate the discrete Fourier transform of the sequence [1,0,0,1] using the decimation in time FFT algorithm. (8)

END OF QUESTION PAPER

GOOD LUCK!

$f(t) \qquad \text{Definition} \qquad \int_{0}^{\infty} f(t)e^{-st} dt$ $Kf(t) \qquad \text{Linearity} \qquad \text{KF(s)}$ $\frac{df(t)}{dt} \qquad \text{Differention} \qquad \text{sF(s)} - f(0)$ $\frac{d^{n} f(t)}{dt^{n}} \qquad \text{Differentiation} \qquad s^{n} F(s) - s^{n-1} f(0) - \dots - \frac{d^{n-1} f(0)}{dt^{n-1}}$ $\int_{0}^{\infty} f(t) dt \qquad \text{Integration} \qquad \frac{1}{s} F(s)$ $tf(t) \qquad \text{Complex differentiation} \qquad -\frac{dF(s)}{ds}$ $e^{-at} f(t) \qquad \text{Complex translation} \qquad F(s+a)$ $f(t-a)u(t-a) \qquad \text{Real translation} \qquad e^{-as} F(s)$ $f(t) \qquad \text{Periodic function} \qquad \frac{F_{1}(s)}{1-e^{-sT}}$ $\int_{0}^{\infty} x(\tau)h(t-\tau) \qquad \text{Convolution} \qquad H(s)X(s)$ $\delta(t) \qquad \qquad 1$ $u(t) \qquad \qquad \frac{1}{s}$ $e^{-at} u(t) \qquad \qquad \frac{\beta}{s^{2}+\beta^{2}}$ $cos \beta tu(t) \qquad \qquad \frac{\beta}{(s+a)^{2}+\beta^{2}}$ $e^{-at} \cos \beta tu(t) \qquad \qquad \frac{s+a}{(s+a)^{2}+\beta^{2}}$ $tu(t) \qquad \qquad \frac{1}{s^{2}}$ $\frac{1}{s^{n}}$	f(t)	TABLE OF LAPLACE TRANS		
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SOME CO	OMMON z-TRANSFORM PAIRS	
Transform pair Signs	al Transform	ROC
1. δ[n]	1	All z
2. u[n]	$\frac{1}{1-z^{-1}}$	z > 1
3. $u[-n-1]$	$\frac{1}{1-z^{-1}}$	z < 1
4. $\delta[n-m]$	Z ^{−m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$ z > \alpha $
$6\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$ z < \alpha $,
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$ z > \alpha $
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$ z < \alpha $
9. $[\cos \Omega_0 n] u[n]$	$\frac{1 - [\cos \Omega_0] z^{-1}}{1 - [2\cos \Omega_0] z^{-1} + z^{-2}}$	z > 1
10. $[\sin \Omega_0 n]u[n]$	$\frac{[\sin \Omega_0] z^{-1}}{1 - [2\cos \Omega_0] z^{-1} + z^{-2}}$	z > 1
11. $[r^n \cos \Omega_0 n]u[n]$	$\frac{1 - [r\cos\Omega_0]z^{-1}}{1 - [2r\cos\Omega_0]z^{-1} + r^2z^{-2}}$	z > r
12. $[r^n \sin \Omega_0 n]u[n]$	$\frac{[r \sin \Omega_0]z^{-1}}{1 - [2r \cos \Omega_0]z^{-1} + r^2z^{-2}}$	z > r

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