NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

BACHELOR OF COMMERCE HONOURS DEGREE

QUANTITATIVE ANALYSIS FOR BUSINESS – CIN 1106

JANUARY 2004 FIRST SEMESTER EXAMINATION

INSTRUCTIONS TO CANDIDATES

1. Answer **all** questions in Section A.
2. Choose and answer three(3) out of five(5) questions in Section B.
3. Answer both questions in Section C.
4. Graph paper will be provided.
5. Statistical tables will be provided.
6. You may use a non-programmable Scientific Calculator.

SECTION A (ANSWER ALL QUESTIONS) [40 MARKS]

Question One

a) Find the equation describing the average rate of change from $x = a$ to $x = a + \Delta x$ for the following functions and evaluate the average rate of change over the given interval

i) $f(x) = 4x^2 + 7$ (a) from $x = 3$ to $x = 5$
(b) from $x = 6$ to $x = 10$
[2; 2; 2 marks
[Total 6 marks]

ii) $f(x) = e^x$ (a) from $x = 1$ to $x = 3$
(b) from $x = 1$ to $x = 4$
[2; 2; 2 marks]
[Total 6 marks]

b) Daily demand for a certain commodity appears to be appropriately described by the function:

$$D(x) = 120 - x^2 \qquad 4 \leq x \leq 10$$

where x is selling price per unit in dollars. Find the average rate of change in demand for a price change from \$5 to \$7. **[2 marks]**

c) An object is dropped from the roof of a 500-metre building. The distance d (in metres) of the object above the ground is given as a function of t (in seconds) by the equation.

$f(t) = 500 - 16t^2$

i) Derive a function describing the instantaneous rate of change in d. **[2 marks]**

ii) Sketch a graph of the function. **[4 marks]**

d) Given

$$A = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & 3 \\ 1 & 6 \end{pmatrix}$$

Verify the statements:

i) A(B + C) = AB + AC **[2 marks]**

ii) (B + C)A = BA + CA **[2 marks]**

(iii) (B + C)A = BA + CA **[2 marks]**

e) Give the General statement of the Linear Programming Maximization Model. **[4 marks]**

f) What is i) An Index number? **[2 marks]**

ii) A one-item Index Number? (Give an example) **2 marks]**
[Total 4 marks]

g) Find the Simple Interest on each of the following loans:

i) $250 at simple interest of 12% from April 15 to April 25. **[2 marks]**

ii) $10 000 at simple interest 15% for 6 months. **[2 marks]**

h) What is the simple interest rate when $800 borrowed for 6 months and paid back as $900? **[2 marks]**

SECTION B. CHOOSE AND ANSWER 3 QUESTIONS ONLY OUT OF 5

Question 2

a) Use the inverse method to solve the following system of equations. (Use the Gauss-Jordan method to find the inverse first)

$x_1 + 2x_2 = 5 - x_3$
$4x_2 = 10 + 3x_1 - 5x_3$
$2x_2 + 6x_3 = 8 + 4x_1$ **[10 marks]**

b) Jason invested a total of $10 000 in 3 different savings accounts. The accounts paid simple interest at an annual rate of 8%, 9% and 7,5% respectively. Total interest earned for the year was $845. The amount in the 9% account was twice the amount invested in the 7.5% account. How much did Jason invest in each account? **[10 marks]**

[Total 20 marks]

Question Three

A firm manufactures 4 products A1, A2, A3 and A4. The products are sold as finished goods to other manufacturers and in addition, certain of the products are used in the assembly of other products. Each unit of A2 requires 2 units of A1 in its assembly. Each unit of A3 requires 3 units of A1 and 4 units of A2. Each unit of A4 requires 1 unit of A1 and 5 units of A3.

The firm has just received an order for 10 units of A1, 15 units of A2, 20 units of A3 and 5 units of A4. What total production should it plan? **[20 marks]**

Question Four

Akim's shop produces 2 types of citizen's-band radios – model B and model C. Each radio must be processed on each of 2 assembly lines. Processing times required are as follows:

	Model B	**Model C**
Assembly line 1	0,5 hours	0,4 hours
Assembly line 2	0,25 hours	0,6 hours

Assembly line 1 will be available for 40 hours each week but, because of maintenance requirements assembly line 2 will be available for only 36 hours. Model B radio yields a profit contribution of $80 per unit sold while model C yields a profit contribution of $60 per unit sold. Demand for the radios far exceeds the production capacity of the plant. How many units of each model should Akim's shop produce in order to maximize profit contribution? Formulate a Linear programming model that can aid in this decision-making process and solve it using the SIMPLEX method.

[20 marks]

Question Five

The following table shows chemical requirements and availabilities at Ntelela's Lawn Care Services.

	Quantity of chemical per container (volume measures)		**Minimum amount required in treatment (volume measures)**
Chemicals required	**Solu-X**	**Phos-Pho-Gen**	
Chemical A	4	2	14
Chemical B	1	1	5
Chemical C	2	3	12
Cost per container	$4	$3	

Letting S = No. of canisters of Solu-X to use
P = No. of canisters of Phos-Pho-Gen to use

Formulate the appropriate cost minimization Linear Program and determine the S and P that minimize cost.

[20 marks]

Question Six

The total cost function for a firm is given by:

$$C(x) = 0{,}001x^3 - 0{,}25x^2 + 50x + 1200$$

where $C(x)$ is cost, in dollars, and x is the number of units produced.

(a) (i) Determine the marginal cost function **[4 marks]**

(ii) What is marginal cost when $x = 150$? $x = 160$ **[2, 2][Total 4 marks]**

(iii) What is the approximate cost of the 101^{st} unit produced? **[2 marks]**

(b) Over what range of values is cost

(i) increasing? **[2 marks]**

(ii) decreasing? **[2 marks]**

(iii) Make a rough sketch of C, the cost function. **[6 marks]**

[Total 20 marks]

SECTION C (COMPLSORY) [40 MARKS]

Question Seven

The price for a certain commodity is given by p = D(x) = 100 – 0,1x where x is the quantity demanded in units, at a price of p dollars per unit. Thus, the total revenue function for sale of the product is R(x) = x(100 – 0,1x) = 100x – 0,1x^2. If, for the same product, variable cost per unit follows a linear function

V(x) = 6,7 + 0,033x and fixed costs are $8 000,

(i) determine the total cost function **[2 marks]**

(ii) determine the total profit function **[4 marks]**

(iii) plot the graphs on the same Co-ordinate Axes.
(You must use the quadratic formula to find the roots of the Quadratic equations.)
[14 marks]
[Total 20 marks]

Question Eight

A Company finds that its annual sales (in thousands of dollars) are a function of the amount of money x spend on television advertising and the amount of money y spent on newspaper and trade journal advertising, where both x and y are expressed in thousands of dollars, as given by:

$$f(x,y) = 1500x + 4800y - 10x^2 - 6y^{2\,-}\ 40xy.$$

The company presently has an advertising budget which allocates $40 000 to television and $25 000 to newspaper and trade journal advertising.

a) Based on the present budget what are the expected annual sales? **[4 marks]**

b) Estimate the effect on annual sales if an additional $1 000 is allocated to television advertising while no change is made in the newspaper and trade journal budget, using partial derivatives. **[8 marks]**

c) Estimate the effect on annual sales if an additional $1 000 is allocated to newspaper and trade journal advertising while no change is made in the television advertising budget. **[8 marks]**
[Total 20 marks]

END OF EXAMINATION PAPER