

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

BACHELOR OF COMMERCE(HONS) DEGREE

QUANTITATIVE ANALYSIS FOR BUSINESS CIN 1106

JULY 2004 SUPPLEMENTARY EXAMINATION

DURATION: 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer **all** questions in Section A.
2. Choose and answer three(3) out of five(5) questions in Section B.
3. Answer both questions in Section C.
4. Graph paper will be provided.
5. Statistical tables will be provided.
6. You may use a non-programmable Scientific Calculator.

SECTION A (COMPULSORY) [40 MARKS]

Question One

- (a) Write the 5 x 5 matrix in which

$$a_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i > j \\ 2 & \text{if } i < j \end{cases} \quad a_{ij} = \begin{cases} i & \text{if } i > j \\ j & \text{if } j > i \\ 0 & \text{if } i = j \end{cases}$$

[6 marks]

- (b) Give the General form of the maximization Linear Programming Model.

[3 marks]

- (c) An investor holds 100 shares of stock A, 50 shares of stock B, 500 shares of stock C and 80 shares of stock D. The dividend rate in dollars on each share of these stocks is as follows:

\$35 on stock A, \$8,50 on stock B, \$3,25 on stock C, and \$250 on stock D.

Find the total dividend income for this investor. (use matrix multiplication)

[3 marks]

- (d) Find the total revenue function, $R(x)$ given:

(i) $R'(x) = 18$

(ii) $R'(x) = 80 - 0,4x$

where x is the number of units sold.

[3 marks, 3 marks]

- (e) The rate of change of demand for a product is given by

$D'(x) = 15x + 210$ where $D'(x)$ is marginal demand in units, and x is the price per unit in \$.

(i) Find the total demand function, $D(x)$ given that $D(0) = 450$.
[4 marks]

(ii) What quantity will be demanded at a price of \$20? \$25?
[2 marks, 2 marks]

(f) Given that

$$A = \begin{pmatrix} 0 & 4 \\ 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 8 \\ 3 & 1 \end{pmatrix}$$

$$x = 4, \quad y = \frac{1}{3}$$

Verify that $xA = Ax$ [2 marks]

$$(x + y)A = xA + yA \quad [2 \text{ marks}]$$

$$x(A + B) = xA + xB \quad [2 \text{ marks}]$$

$$x(yA) = (xy)A \quad [2 \text{ marks}]$$

(g) Finance House I offers 16,75% p.a. compounded quarterly and Finance II offers 16,25% compounded monthly. Which Finance House should we invest in and why?
[2 marks]

(h) Minimize $z = 3x + 2y$
subject to $x + y \geq 20$
 $x + 2y \geq 30$
 $y \geq 5$
 $x, y \geq 0$
[4 marks]

SECTION B (60 MARKS)

Choose and answer three (3) questions ONLY, out of five (5).

Question Two

(a) A firm wishes to establish a sinking fund for the purpose of expanding facilities at its West Nicholson plant. The Company needs to accumulate \$1 600 000.00 over the next 5 years. A conservative investment policy could earn 16% p.a. compounded quarterly. How much should the firm contribute to the fund every four months in order to accumulate the desired amount in 5 years?
[5 marks]

(b) How much will have to be deposited in a fund at the end of each year for 5 years at 12% p.a. compounded annually to pay off a debt of \$1 500 000.00.
[5 marks]

Construct the appropriate sinking fund schedule. [10 marks]

[Total 20 marks]

Question Three

- (a) According to the ideal-gas law for a confined gas, if P is the number of kg per square unit in the pressure, V is the number of cubic units in the volume, and T is the number of degrees in the Temperature, we have the formula $P.V = k.T$ where k is a constant of proportionality . Show that:

$$\frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} \cdot \frac{\partial P}{\partial V} = -1 \quad \text{[8 marks]}$$

- (b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $z = \frac{xy}{(x^2 - y^2)^2}$

[8 marks]

- (c) A firm’s marginal revenue function is given by $f(x) = -12x + 120$, where x is the number of units produced. The marginal cost function is given by $g(x) = 30$. Find the change in profit when the production level changes from 12 units to 24 units.

[4 marks]

[Total 20 marks]

Question 4

The data below shows a company’s wage bills according to employee category.

EMPLOYEE CATEGORY	NO. OF EMPLOYEES			WAGE BILL (\$000)		
	1990	1991	1992	1990	1991	1992
Unskilled	120	80	60	24	32	36
Semi-skilled	80	100	90	16	20	18
Skilled	20	40	50	30	80	120

1990 = 100

Calculate Fisher’s ideal price index numbers for 1991 and 1992.

[20 marks]

Question Five

- (a) Using the Gaussian method, find the Inverse of the matrix below if it exists:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{pmatrix}$$

[10 marks]

- (b) Using matrix methods solve the following system of linear equations:

$$2x + 6y - z = 18$$

$$y + 3z = 9$$

$$3x - 5y + 8z = 4$$

[10 marks]

[Total 20 marks]

Question Six

Using Geometric Progressions, derive the formula for :

i) Present value of an Ordinary Annuity. [10 marks]

ii) Future value of an Ordinary Annuity. [10 marks]

[Total 20 marks]

SECTION C (COMPULSORY) (40 MARKS)

Question Seven

The following table relates to a 3 Industry Economy and figures are in \$billion.

PRODUCER	USER			FINAL CONSUMER DEMAND
	Agriculture	Industry	Transport	
Agriculture	12	12	5	31
Industry	8	24	10	38
Transport	5	16	8	11

If Consumer Demand for agriculture is predicted to increase to 35, Industry to 40 and transport to 12, what is the required level of output to satisfy the new demand?

[20 marks]

Question Eight

The price for a certain product is given by:

$P = D(x) = 100 - 0,1x$ where x is the quantity demanded, in units, at a price of p dollars per unit. Thus the total revenue function for the sale of the product is

$$\begin{aligned} R(x) &= x (100 - 0,1x) \\ &= 100x - 0,1x^2. \end{aligned}$$

If variable cost for this product $v(x) = 6,7 + 0,033x$ and fixed costs are \$8 000, determine:

(a) the total cost function. [5 marks]

(b) the profit function. [5 marks]

(c) sketch the 3 graphs on the same co-ordinate axis and show the break even points, i.e. where Total Revenue = Total Cost. [10 marks]

[Total 20 marks]

END OF EXAMINATION