NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM HONOURS DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS 1 (CIN 2110)

December 2002 1ST SEMESTER EXAMINATION

INSTRCTION TO THE CANDIDATES

a) **Answer all questions**
b) **In addition to this paper you require Actuarial tables and Electronic calculator.**

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1 The price of a given share is 80 cents. The risk-free rate of interest is 5% per annum convertible quarterly. Assuming no arbitrage and that the share will not pay any income, calculate the forward price for the share, for settlement in exactly one quarter of a year.   **[2 marks]**

2 An investor purchases a bond on the issue date at a price of $96 per $100 nominal. Coupons at an annual rate of 4% are paid annually in arrears. The bond will be redeemed at par twenty years after the issue date.

Calculate the gross redemption yield from the bond.   **[4 marks]**

3 An ordinary share pays annual dividends. The next dividend is expected to be 10 cents per share and is due in exactly 9 months time. It is expected that subsequent dividends will grow at a rate of 5% per annum compound and that inflation will be 3% per annum. The price of the share is 250 cents and dividends are expected to continue in perpetuity.

Calculate the expected effective real rate of return per annum for an investor who purchases the share.   **[5 marks**]

4 A loan of $ 10 000 is repayable by 36 monthly installments in arrear of $320. Calculate the APR on the transaction.   **[5 marks]**

5   (a) State the characters of property investments   **[3 marks]**

       (b) Distinguish a repayment loan from an interest only loan   **[3 marks]**

                                             [Total 6 marks]

6   In return for a loan of \$ 15 000 at t=0 a bank will receive repayments of (19+X) at times t=1, 2, …, 20, 40X at times t=21, 22, …, 40. The contract rate of interest is 12% per unit time. What is the value of X?   **[6 marks]**

7   The forward rate from time $t = 1$ to time t $f_{t=1,t}$ has the following values:

$f_{0,1} = 4.0\%$, $f_{1,2} = 4.5\%$, $f_{2,3} = 4.8\%$.

(i) Assuming no arbitrage, calculate:

(a) the price per \$100 nominal of a 3-year bond paying an annual coupon in arrears of 5%, redeemed at par in exactly three years, and

(b) the gross redemption yield from the bond   **[6 marks]**

8   The following investments were made on 15 January 1993.

Investment A: £10,000 was placed in a special savings account with a 5-year term. Invested money was accumulated at 3½% per annum effective for the first year, 4½% per annum effective for the second year, 5½% per annum effective for the third year, 6½% per annum effective for the fourth year and 7½% per annum effective for the fifth year.

Investment B: £10,000 was placed in a zero coupon bond with a 5-year term in which the redemption proceeds were the amount invested multiplied by the ratio of the Retail Price Index for the month two months prior to that in which redemption fell to the Retail Price Index for the month two months prior to the date of investment; this amount was further increased by 2¾% compound for each complete year money was invested.

Investment C: An annuity, payable annually in arrears for 5 years was purchased for £10,000 to yield 6¼% per annum effective.

The Retail Price Index at various times was as follows:

November 1992	237.6
January 1993	240.0
January 1994	250.0
January 1995	264.4
January 1996	266.6
January 1997	270.4
November 1997	274.0
January 1998	275.6

(i)   What is the amount of the annual income yielded by the annuity?

[1 mark]

(ii)   Calculate the real rate of return per annum earned on investment A and on nvestment B over the period 15 January 1993 to 15 January 1998. The investor is not liable for tax.   **[4 marks]**

(iii)   Determine which of the three investments yielded the highest real rate of return per annum over the period 15 January 1993 to 15 January 1998.

[3 marks]

[Total 8 marks]

9 The force of interest $\delta(t)$ at time t is given by

$$\delta(t) = \begin{cases} 0.1 - 0.005t ....t < 100 \\ 0.05 ....t > 10 \end{cases}$$

Calculate:

(a) The accumulation at time $t = 10$ of $ 1000 invested at time t=0   **[2 marks]**

(b) The equivalent constant rate of interest per annum convertible quarterly which would give the same accumulation as in (a)   **[3 marks]**

(c) The present value at time t=10 of a continuous payment stream at the rate of $100e^{0.1t}$ to time t=20.   **[4 marks]**

[Total 9 marks]

10 (a) Show that:

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$$\sum t^2 v^t = \frac{2(Ia)_{\overline{n}|} - a_n - n^2 v^{n+1}}{d}$$ **[4 marks]**

(b) One year's payment is $5000 and each subsequent year's payment is $100 more than the previous year. Using an interest rate of 7% per annum, find the discounted mean term of this annuity. **[5 marks]**

[Total 9 marks]

11 The yields on a fund in different years are independently and identically distributed. Each year the distribution of $(1+ i_t)$ where i_t is the rate of interest earned in the year t, is log-normal with parameters μ and σ^2.

If i_t has a mean value of 0.09 and standard deviation 0.06,

(a) Calculate the parameters μ and σ^2 **[4 marks]**

(b) Calculate the expected value and standard deviation of the accumulation of $1000 at the end of five years **[3 marks]**

(c) Calculate the probability that the accumulation in (b) is less than $1300

[3 marks]

[Total 10 marks]

12 A loan of $10 000 is repayable by a level annuity payable annually in arrears for 20 years. The repayments were calculated using a rate of interest of 9% per annum effective for the first 8 years and 12% per annum thereafter.

At the end of eight years immediately before the ninth repayment due, it is agreed that the interest rate on the loan will not increase but will remain 9% per annum effective. It is also agreed that the annual repayment will continue unchanged, payable for a revised term but with a final reduced payment.

(a) Calculate the level annual repayment **[3 marks]**

(b) Calculate the total amount of interest payable in the first eight payments.

[3 marks]

(c) Calculate the total revised term of the loan **[4 marks]**

(d) Calculate the amount of the final repayment **[2 marks]**

[Total 12 marks]

13 (a) What are the usual purposes for which discounted cash flow techniques are used?

[2 marks]

(b)What is meant by the internal rate of return in a discounted cash flow calculation?

[2 marks]

(c) A company is considering two mutually exclusive projects as follows:

a. Project A involves capital costs of $ 100 000 at the beginning of each of the first three years. The income is expected to commence at a level of $20 000 per annum after 4 years and to be payable continuously for 17 years. Once begun, the income is expected to increase on each anniversary of commencement of the project to a level of 10% greater than the preceding year.

b. Project B requires an immediate capital payment of $ 300 000 and is expected to return income of $ 64 000 at the end of 2, 4, 6 years and so on, ending with the income received at the end of the 20th year.

   I. What is the internal rate of return, on each project **[11 marks]**

   II. Apart from the internal rate of return, what other factors might the company consider before deciding to go ahead with one of the projects **[3 marks]**

[Total 18 marks]

END OF EXAMINATION