NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM (HONOURS) DEGREE ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS – CIN 2110

JULY 2003 SUPPLIMENTARY EXAMINATION

DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer all questions.
2. You must not start writing your answers in the booklet until instructed to do so by the invigilator
3. Answer to each question on a separate sheet.
4. In addition to this question paper you should have an electronic calculator and actuarial tables

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1. The following n-year spot rates apply at time t = 0

1 year spot rate of interest:	4½% per annum effective
2 year spot rate of interest:	5% per annum effective
3 year spot rate of interest:	5½% per annum effective

    Calculate the two year forward rate of interest from time t = 1 expressed as an annual effective rate of interest. [2 marks]

2. Describe what is meant by a "currency swap". [3 marks]

3. For a rate of interest of 7% per annum, convertible monthly, calculate:

    (i) the equivalent rate of interest per annum convertible half yearly, and
    [2 marks]

    (ii) the equivalent rate of discount per annum convertible monthly
    [2 marks]
    [Total 4 marks]

4. A certain company issues both debenture stocks and unsecured loan stocks. Describe the differences between these two types of asset. [4 marks]

5. You are given the following information in respect of a pension fund:

Calendar Year	Value of fund at 1 January	Value of fund at 30 June	Net cash flow received on 1 July
1997	£180,000	£212,000	£25,000
1998	£261,000	£230,000	£18,000
1999	£273,000	£295,000	£16,000
2000	£309,000		

Calculate the annual effective time weighted rate of return earned on the fund over the period from 1 January 1997 to 1 January 1997 to 1 January 2000.

[4 marks]

6.   (i)   Calculate $\ddot{s}^{(12)}_{\overline{5.5}}$ at an effective rate of interest of 13% per annum.

[3 marks]

      (ii)   Explain what your answer to (i) represents.   [2 marks]

[Total 5 marks]

8.   The force of interest is given by:

$$\delta(t) = 0.05 + 0.001t + 0.0001t^2 \qquad 0 \leq t \leq 10$$

      (i)   Calculate the total at time 10 of the accumulated proceeds of an investment of £100 at time 5.   [5 marks]

      (ii)   Calculate the equivalent constant force of interest earned on the transaction.   [3 marks]

[Total 8 marks]

9.   Describe the main differences for an investor between equity shares and convertible debentures issued by a typical company.   [4 marks]

10.   Distinguish between the terms in each of the following pairs, in the context of derivative markets.

      (a)   long party and short party
      (b)   call option and put option   [3 marks]

11.   An investor is considering two investments. One is a 3-month deposit account which pays a rate of return of 4% p.a. convertible half yearly. The second is a 3-months Government Bill. Calculate the annual simple rate of discount available from the government Bill if both investments provide the same effective rate of return.   [3 marks]

12.   An investment fund is valued at £1.2 million on 1 January 1996 and at £1.4 million of 25 March 1996. Immediately after the second valuation a payment of

£0.2 million is paid into the fund. On 31 December 1996 the fund is valued at £1.8 million. Calculate the time weighted rate of return for the year.

[2 marks]

13. A customer takes out a loan of $1 000 in a consumer credit transaction. The loan is repaid by instalments of $703.84 p.a. payable annually in arrear for two years. What is the APR for the transaction? [5 marks]

14. $1 000 accumulate $1 100 in nine months.

Calculate :
    (a)   The annual simple discount rate
    (b)   The effective annual compound discount rate.   [5 marks]

15. A borrows $100 from B, to be repaid with accrued interest one year later. How much is A's payment to B if B charges:

a)   An interest rate of 9.5% convertible half yearly?
b)   A discount rate of 9% per annum?
c)   A discount of 9% per annum convertible half yearly
d)   A force of interest of 9.5 % per annum?   [Total 10 marks]

16. In a consumer credit transaction, an individual borrows $2 000 to be repaid by level monthly installments in arrear over three years. The repayments are calculated on the basis of a flat rate of interest of 13.75% per annum. Calculate:

a)   The monthly premium   [3 marks]
b)   The effective rate of interest paid by the individual   [4 marks]
c)   The APR for the transaction   [3 marks]
                                                         [Total 10 marks]

17. In any year, the rate of interest on funds invested with a given insurance company is independent of the rates of interest in all precious years.

Each year the value of $(1 + i_t)$, where I_t is the rate of interest earned in the t^h year, is lognormally distributed. The mean and standard deviation of I_t are 0.07 and 0.20 respectively.

(i)   Determine the parameters ③ and ⑨$_2$ of the lognormal distribution of $1 + i_t$.
                                                         [6 marks]
(iii)   a) Determine the distribution of S_{15}, where S_{15} denotes the accumulation of one unit of money over 15 years.

       b) Determine the probability $S_{15} > 2.5$.   [4 marks]
                                                         [Total 10 marks]

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18. A pension fund expects to make payments of £100,000 per annum at the end of each of the next five years. It wishes to immunise the liabilities by investing in two zero coupon bonds which mature in five years and in one year respectively. The rate of interest if 5% per annum effective.

i)   a) Show that the present value of liabilities is £432,948.
       b) Show that the duration of the liabilities is 2.9 years.

ii)   Calculate the nominal amounts of the two zero coupon bonds which must be purchased if the pension fund is to equate the present value and duration of assets and liabilities.

iii)   a) Calculate the convexity of the assets.

       b) Without calculating the convexity of the liabilities, comment on whether you think Redington's immunisation has been achieved.
                                                                        [6 marks]
                                                                  [Total 18 marks]

END OF EXAMINATION PAPER