**NATIONAL UNIVERSITY OT SCIENCE AND TECHNOLOGY**

**B. COMM HONOURS DEGREE IN ACTUARIAL SCIENCE**

**ACTUARIAL MATHEMATICS 1 : CIN 2110**

**FIRST SEMESTER EXAMINATIONS : JANUARY 2004**

**INSTRCTION TO THE CANDIDATES**

1. **Answer all questions**
2. **Answer each question on a new sheet.**
3. **For this question paper you are permitted to have an electronic calculator (non-programmable) and actuarial tables**
4. **You must not start writing your answers until instructed to do so by the invigilator.**

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1. In the context of Financial Mathematics describe briefly by way of examples, the concept of a "swap" **[3 marks]**

2. Calculate the effective rate of interest per annum equivalent to each of the following rates:
   (i) An effective rate of discount of 6% per annum.
   (ii) A nominal rate of interest of 6% per annum convertible monthly
   (iii) A nominal rate of discount of 6% per annum convertible quarterly
   (iv) An effective rate of interest of 6% per half-year
   (v) A force of interest of 6% per annum
   (vi) A force of interest of 6% per half year

   **[Total 6 marks]**

3. Derive the formula:

   (i) $\sum_{t=1}^{n} tv^t = \frac{a_n - nv^n}{i}$ where $v = (1+i)^{-1}$ and $i > 0$ **[2 marks]**

   (ii) $(\bar{I}\bar{a})_n = \frac{\bar{a}_n - nv^n}{\delta}$ where $\delta > 0$ **[3 marks]**

   **[Total 5 marks]**

4. Given an effective rate of 7% per annum; determine the present value of each of the following cash flows:

   (i) Payments of $30 per quarter, payable quarterly in advance for 20 years. **[2 marks]**

   (ii) Payments of $30 per quarter payable quarterly in arrear for a total of 15 payments. **[3 marks]**

   (iii) Payments of $60 payable at the end of every second year for a total of 10 payments. **[3 marks]**

   (iv) A payment of $100 per annum payable continuously during year 1, increasing to $150 payable continuously during year 2, $200 payable continuously during year 3 and so on, with a final payment of $550 payable continuously during year 10. **[4 marks]**

   (v) A payment of $10 at the end of the first quarter, increasing by $10 at the end of each subsequent quarter for a total of 40 payments. **[5 marks]**

   **[Total 17 marks]**

5. (i) Explain the term "Arbitrage" **[2 marks]**

   (ii) In light of the uncertainty in the availability of maize grain from Zimbabwe Grain Marketing Board, the current price of maize grain per bucket is $4500 and the risk-free force of interest is 7% per annum. Storing maize grain costs you $200 per bucket per year payable at the end of the year. Derive the forward price for buying a bucket of maize grain in one year's time by applying an arbitrage argument. **[4 marks]**

6. One-to-One insurance company offers loans to their life policy holders. If a loan is given, it has to be repaid by an immediate annuity. The annuity starts at a rate of $100 per annum and increases by $10 per annum. The annuity is paid for 20 years. Repayments are calculated using a rate of interest of 8% per annum effective.

   (i) Calculate the amount of the loan. **[2 marks]**

   (ii) Construct a loan schedule showing the capital and interest elements in the amount of loan outstanding after the $6^{th}$ and $7^{th}$ payments. **[4 marks]**

   (iii) Find the capital and interest element of the last instalment. **[2 marks]**

   **[Total 8 marks]**

7. Payments are made continuously into an account such that the rate of payment at time t years is $\$(100 - 5t)$ per annum. The account accumulates continuously in

such a way that the force of interest at time t years is $\delta(t) = 0.1 - 0.005t$. Given that payments start at time t = 0 , find the amount of the account after 20 years.

**[6 marks]**

8. Let $i_t$ denote the interest rate in year t. It is assumed that for each year $t$, $i_t$ will be

$$i_t = \begin{cases} 0.03 & \text{with probability } \frac{1}{3} \\ 0.07 & \text{with probability } \frac{2}{3} \end{cases}$$

Calculate

(i) the expected value of $(1 + i_t)$ **[2 marks]**

(ii) the standard deviation of $(1 + i_t)$ **[3 marks]**

**[Total 5 marks]**

9. (i) In the context of project appraisals define the "discounted payback period".

**[2 marks]**

(ii) Max Trading borrows \$10 000 at an effective rate of interest of 15% per annum to finance their Victoria Falls project. This project will bring an income at a level rate of \$1800 per annum payable quarterly in arrear for 20 years. Calculate the discounted payback period. **[4 marks]**

**[Total 6 marks]**

10. (i) Explain briefly the concept of immunization in the context of the assets and liabilities of a life office. **[2 marks]**

(ii) State three conditions necessary for applying Redington's theory of immunization. **[3 marks]**

(ii) Check-Master insurance company has issued a number of policies for which a total liability of \$1 000 000 is payable in exactly 10 years time. The company holds sufficient funds to cover the liability on the basis of a force of interest of $\delta$ per annum and intends to invest this money solely in the purchase of the following types of bond:

A : Zero-coupon bonds redeemable at par in 20 years time.
B: Zero-coupon bonds redeemable at par in 5 years time.

What proportion of the company's funds should be invested in the 5 year – zero coupon bonds to ensure that the discounted mean term of the assets equals the discounted mean term of the liabilities at a constant force of interest of $\delta$ per annum given that this is also the market basis for pricing all stocks? **[6 marks]**

**[Total 11 marks]**

11. The n-year forward rate for transactions beginning at the time t and maturing at time t + n is denoted as $f_{t,n}$. You are given

$f_{0,1}$ = 6.0% per annum
$f_{0,2}$ = 6.5% per annum
$f_{1,2}$ = 6.6% per annum

Determine the 3-year par yield. **[3 marks]**

12. A loan of nominal amount \$100 000 is to be issued bearing interest payable quarterly in arrear at a rate of 8% per annum. Capital is to be redeemed at \$105% on a coupon date between 15 and 20 years after the date of issue, inclusive, the date of redemption being at the option of the borrower.

(i) An investor who is liable to income tax at 40% and tax on capital gains at 30% wishes to purchase the entire loan at the date of issue.

What price should she pay to ensure a net effective yield of least 6% per annum? **[6 marks]**

(ii) Exactly 10 months after issue the loan is sold to an investor who pays income tax at the 20% and capital gains tax at 30%.
Calculate the price this investor should pay to achieve a yield of 6% per annum on the loan:

(a) assuming redemption at the earliest possible date. **[3 marks]**

(b) assuming redemption at the latest possible date. **[3 marks]**

**[Total 12 marks]**

13. In order to assess investment performances between KB and KD Fund Managers, an investor placed part of his assets with each of them. Values of the funds for the period 1 January 2000 to 31 December 2002 for each manager were as given below:

| | **KB** | **KD** |
|---|---|---|
| 01.01.2000 | \$120 000 | \$100 000 |
| 31.12.2000 | \$130 000 | \$140 000 |
| 31.12.2001 | \$135 000 | \$145 000 |
| 31.12.2002 | \$180 000 | \$150 000 |

You are also advised that on 1 January 2001 \$10 000 was subsequently invested with each fund manager and a further \$10 000; was invested on 1 January 2002. (**Note**: The fund values above excludes these cashflows):

(i) Calculate the time-weighted rates of return earned by each fund manager over the period 1 January 2000 to 31 December 2002. **[4 marks]**

(ii) Calculate the money-weighted rate of return earned by KB fund managers over the period 1 January 2000 to 31 December 2002. **[3 marks]**

(iii) Without calculating the money-weighted rate of return for KD fund managers state with reasons whether the money-weighted rate of return earned by KD fund managers over the same period is equal, lower or higher than that of KB fund managers. **[2 marks]**

(iii) Comment briefly on the relative performance of the two fund managers. **[3 marks]**

**[Total 12 marks]**

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**END OF EXAMINATION PAPER**