NATIONAL UNIVERSITY OT SCIENCE AND TECHNOLOGY

B. COMM HONOURS DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS 1 : CIN 2110

JULY 2004 SUPPLIMENTARY EXAMINATIONS

INSTRCTION TO THE CANDIDATES

1. **Answer all questions**
2. **Answer each question on a new sheet.**
3. **For this question paper you are permitted to have an electronic calculator (non-programmable) and actuarial tables**
4. **You must not start writing your answers until instructed to do so by the invigilator.**

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1. In the context of derivative markets, describe briefly the following terms:

   (i) A future

   (ii) Long term

   (iii) Short-part

   [6 marks]

2. State the main differences between a preference share and an ordinary share.
   [4 marks]

3. Zidco Insurance company calculates the single premium for a contract paying $500000 in five years time as the present value of the benefit payable at the expected rate of interest it will earn on its invested funds. The annual effective rate of interest over the whole of the next five years will be 6%, 7% or 9% with probabilities 0.3, 0.5 and 0.2 respectively.

   (a) Calculate the single premium. **[3 marks]**

   (b) Calculate the expected profit at the end of the term of the contract.
   [4 marks]
   [Total 7 marks]

4. A loan of $100000 is to be repaid by equal instalments at the end of each month for 20 years. The amount of the instalment is calculated on the basis of an interest rate of 8% per annum effective.

   (i) Calculate the amount of the monthly instalment. **[3 marks]**

(ii) Calculate the amount of the loan outstanding at the start of the 11^{th} year. **[3 marks]**

(iii) Calculate the total amount of interest paid in the 11^{th} year. **[5 marks]**
[Total 1 marks]

5. Given $i = 6.2\%$
Evaluate $a_{10}^{(4)}$ **[3 marks]**

6. A loan is to be repaid by an increasing annuity payable annually in arrear for ten years. The first payment of the nnuity will be \$500 and payments will increase by \$100 each. Ignoring taxation the lender calculates that he will obtain a yield of 5% per annum effective.

Find the amount of the loan. **[6 marks]**

7. A bond is to be issued bearing a coupon of c% per annum payable m times a year in arrear and maturing in n years. The redemption price if R (per \$100 nominal). An investor not subject to tax purchases this bond for the price R per \$100 nominal.

Calculate the yield i per annum effective the investor will obtain, and calculate i^{m} .
[7 marks]

7. On 1 January 1999 you invested \$100 000 with your investment manager. He advised you on 30 June 1999 that your fund was worth 135 000 and you invested a further \$75 000 with him on 1 July 1999. On 31^{st} December 1999 you are advised that your fund was worth \$205 000.

(i) Calculate your money-weighted rate of return per annum. **[4 marks]**

(i) Calculate the time weighted rate of return per annum achieved by your investment manager. **[4 marks]**
[Total 8 marks]

9. You are given the following values for the Retail Prices Index (RPI)

Date	**RPI**
March 1996	151.5
March 1997	155.4
March 1998	160.8
March 1999	164.1
March 2000	168.4
September 2000	171.7

(i) Calculate the average annual rate of inflation between March 1996 and March 2000 **[2 marks]**

(ii) A given set of goods costs $15 000 in March 1998.

(a) Estimate the cost of this set of goods in March 1996 **[3 marks]**
(b) Estimate the cost of this set of goods in September 2000. **[3 marks]**

[Total 8 marks]

10. Consider two fixed interest securities, stock A and stock B. For each of these securities the coupon rate is 8% per annum, dividends are payable annually and the next dividend is payable one year from now. For both securities the redemption price is $100%. Stock A will be redeemed in three years and stock B will be redeemed in 10 years.

Assuming that the market rate of interest for valuing these stocks is 6% per annum effective, calculate the price of each stock per $100 nominal. **[6 marks]**

11. At time 0, the 1-year spot rate is 8% per annum, the 2-year spot rate is 9% per annum and the 3-year spot rate is 9½% per annum.

(i) Define what is meant by an n-year spot rate of interest. **[2 marks]**

(ii) Calculate the value of the 2-year forward rate from time T. **[4 marks]**

[Total 6 marks]

12. Describe briefly what is meant by:

(i) a "forward contract" **[2 marks]**
(ii) "short-selling" **[2 marks]**
(iii) "no arbitrate assumption" **[2 marks]**

[Total 6 marks]

13. A bond has a term of 20 years with a coupon rate of 8% per annum and payable quarterly. The redemption price per $100 nominal is $110. An investor who wishes to obtain a yield of 5% per annum effective (after taxation) purchased this bond.

(i) Calculate the purchase price per $100 nominal if the investor is not subject to tax? **[2 marks]**
(ii) Calculate the purchase price per $100 nominal if the investor is subject to 40% on income tax and no capital gains tax. **[3 marks]**
(iii) Calculate the purchase price per $100 nominal if the investor is subject to 40% income tax and 30% CGT. **[5 marks]**

[Total 10 marks]

14. The force of interest $\delta(t)$ is a function of time, t (measured in years) and at any time t is given by the formula

$$\delta(t) = \frac{0{,}05}{1 + 0{,}05t}$$

(i) Obtain the present value (as a function t) of T unit due at time t

[4 marks]

(ii) Calculate $(\bar{I}\bar{a})_{10}$, where the present value is evaluated at time, $t = 0$

[8 marks]

[Total 8 marks]

END OF EXAMINATION PAPER