NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM (HONOURS) ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS I : CIN 2110

NOVEMBER/DECEMBER 2004 FIRST SEMESTER EXAMINATION

3 HOURS

INSTRUCTIONS TO CANDIDATES

1. **Answer all questions**
2. **Answer each question on a new sheet.**
3. **For this question paper you are permitted to have an electronic calculator (non-programmable) and actuarial tables**
4. **You must not start writing your answers until instructed to do so by the invigilator.**

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1. Describe briefly what is meant by:

   (i) a "forward contract" **[1 marks]**
   (ii) short-selling **[1 marks]**
   (iii) no arbitrage assumption **[2 marks]**

   [Total : 4 marks]

2. A financial institution offers a loan of \$3 000. The loan is to be repaid over a four year term by level monthly installments of \$80, payable in arrears. What is the APR for this transaction. **[5 marks]**

3. The interest on a bank account is calculated as follows:

   during the first year (i.e. year 1), using 6% per annum nominal , convertible monthly,
   during year 2, using a rate of interest which is the equivalent of a rate of discount of 8% per annum nominal, convertible quarterly.
   During year 3 and the first half of year 4, using a force of interest of 7% per annum.
   During the second half of year 4 and during the whole of year 5, using a rate of interest of 6% effective per half-year,
   During year 6, using a force of interest where the force of interest per annum at time t, is $\delta(t) = 0.05 + 0.02\ (t - 5)$ for $5 \leq t \leq 6$

   Determine the amount to which \$100 would accumulate if placed in the above bank account at the start of the first year and withdrawn at the end of year 6.
   [8 marks]

4. A loan of \$100 000 is to be repaid by equal installments at the end of each month for 20 years. The amount of the installment is calculated on the basis of an interest rate of 8% per annum effective.

(i) Calculate the amount of the monthly installment. **[2 marks]**

(ii) Calculate the amount of the loan still outstanding at the start of the 11^{th} year. **[3 marks]**

(iii) Calculate the total amount of interest paid in the 11^{th} year. **[5 marks]**

[Total : 10 marks

5(a) Discuss why the internal rate of return might not be an appropriate tool for the comparison of two projects. **[3 marks]**

(b) Define what is meant by "discounted payback period". **[2 marks]**

(c) Give a verbal definition for each of the following terms:

(i) an n-year zero coupon bond. **[1 mark]**
(ii) a forward rate **[1 mark]**
(iii) n-year par yield **[1 mark]**

[Total :8 marks]

6. Show that:

(i) $\sum_{t=1}^{n} t^2 v^t = \frac{2(Ia)_n - a_n - n^2 v^{n+1}}{d}$ **[4 marks]**

(ii) $(\bar{I}\,\bar{a})_n = \frac{\bar{a}_n - nV^n}{\delta}$ **[3 marks]**

[Total : 7 marks]

7. A fund had a value of \$100 000 on 1 July 1993. A net cash flow of \$20 000 was received on 1 July 1994 and a further net cash flow of \$35 000 was received on 1 July 1995. Immediately before receipt of the first cash flow the fund had a value of \$115 000 and immediately before the second cash flow the fund had a value of \$155 000. The value of the fund on 1 July 1996 was \$180 000.

(i) Calculate the money weighted rate of return earned on the fund between 1 July 1993 and 1 July 1996. **[3 marks]**
(ii) Calculate the time weighted rate of return earned on the fund between 1 July 1993 and 1 July 1996. **[3 marks]**
(iii) Comment briefly on why these two values differ. **[2 marks]**

[Total : 8 marks]

8. A business venture requires an initial investment of \$10 000 and a further investment of \$3 000 in a year's time. The venture will produce an income of \$500 in two years' time; \$1000 in three years' time, \$1500 in four years' time and so on, the final income being \$4 000 in nine years' time.

(i) Find the internal rate of return for this project. **[4 marks]**

(ii) An investor has no spare cash, but may borrow money at any time at the fixed rate of 5% per annum, all loans being repayable in whole or in part at any time at the borrower's discretion. The investor is considering borrowing to finance the venture described above. Should he undertake the venture and if so what will his profit be in nine years' time on the completed transaction?

[6 marks]
[Total : 10 marks]

9. The following n-year spot rates were observed at time t=0.

1 year spot rate of interest	4%
2 year spot rate of interest	5%
3 year spot rate of interest	6%
4 year spot rate of interest	7%
5 year spot rate of interest	7.5%
6 year spot rate of interest	8%

(i) Calculate $f_{3,2}$ **[5 marks]**

(ii) Using the above n-year spot rates calculate the 6-year par yield at time t=0. **[3 marks]**
[Total : 8 marks]

10. The rate of interest i_t in any year has a mean of 6% and standard deviation of 1%. The yield in any year is independent of the yields in all previous years.

(i) Find the standard deviation of the accumulated value at time 12 of an investment of \$10 at time t = 0. **[6 marks]**

(ii) Given that each year the value of $(1 + i_t)$ is log-normally distributed, (where i_t is the rate of interest earned in the t^{th} year):

Find the parameters μ and σ^2 for the log-normal distribution of $(1 + i_t)$
[4 marks]
[Total : 10 marks]

11. A bond has a term of 20 years with a coupon rate of 8% per annum and payable quarterly. The redemption price per $100 nominal is $110. An investor who wishes to obtain a yield of 5% per annum effective (after taxation) purchased this bond.

(i) Calculate the purchase price per $100 nominal if the investor is not subject to tax? **[3 marks]**

(ii) Calculate the purchase price per $100 nominal if the investor is subject to 40% on income tax and no capital gains tax. **[4 marks]**

(iii) Calculate the purchase price per $100 nominal if the investor is subject to 40% income tax and 30% capital gains tax? **[5 marks]`**

[Total : 12 marks]

12. An investor has to pay a lump sum of $20 000 at the end of fifteen years from now and an annuity of $5 000 per annum payable half-yearly in advance for twenty-five years, starting in ten years time. The investor currently holds an amount of cash equal to the present value of these two liabilities valued at an effective rate of interest of 7% per annum. The investor wishes to immunize her fund against small movements in the rate of interest by investing the cash in two zero-coupon bonds, bond X and bond Y. The market prices of both bonds are calculated at an effective rate of interest of 7% per annum. The investor has decided to invest an amount in bond X sufficient to provide a capital sum of $25000 when bond X is redeemed in ten years time. The remainder of the cash is invested in bond Y.

(i) State the conditions necessary for her to be fully immunized in the sense of Redington's immunization theory. **[3 marks]**

(ii) In order to immunize her holdings calculate the amount of money invested in bond Y. **[7 marks]**

[Total : 10 marks]

END OF EXAMINATION PAPER!!!