

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM (HONOURS) ACTUARIAL SCIENCE

ACTUARIAL MATHEMATICS I : CIN 2110

JULY 2005 SUPPLEMENTARY EXAMINATION

DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer all questions
 2. Answer each question on a new sheet.
 3. For this question paper you are permitted to have an electronic calculator (non-programmable) and actuarial tables
 4. You must not start writing your answers until instructed to do so by the invigilator.
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1. Explain what is meant by:

- (i) certificate of deposit [2 marks]
- (ii) short-forward position in a forward contract [2 marks]
- (iii) hedge portfolio [2 marks]

[Total : 6 marks]

2. The rate of discount per annum convertible quarterly is 8%. Calculate:

- (i) the equivalent rate of interest per annum convertible half-yearly. [3 marks]
- (ii) The equivalent rate of discount per annum convertible monthly. [3 marks]

[Total : 6 marks]

3. A businessman borrows \$8 000 from the bank at a rate of interest of 7% effective per annum. He invests this money in a project. The project provides revenues of \$1 000 at the end of each of the first 10 years followed by revenues of \$500 per annum payable continuously for the next 6 years (i.e. during years 11 to 16). The businessman uses the revenues to repay the bank until such time as the loan and interest are fully repaid. Once the loan and interest are repaid, the business invests the revenues in a bank account at an effective rate of interest of 5% per annum.

Determine:

- (i) The amount of the outstanding loan at the end of 10th year just after the revenue then payable has been received. **[5 marks]**
- (ii) The time at which the loan and interest have been fully repaid. **[5 marks]**
- (iii) The net accumulated revenues at the end of the 16th year. **[3 years]**
[Total 13 marks]

4. You are given the following values for the Retail Prices Index::

Date	RPI
March 1996	151.5
March 1996	155.4
March 1998	160.8
March 1999	164.1
March 2000	168.4
September 2000	171.7

- (i) Calculate the average annual rate of inflation between March 1996 and March 2000. **[3 marks]**
- (ii) A given set of goods costs \$15 000 in March 1998.
 - (a) Estimate the cost of this set of goods in March 1996. **[2 marks]**
 - (b) Estimate the cost of this set of goods in September 2000. **[2 marks]****[Total : 7 marks]**

5. Find the present value of an annuity of \$160 per annum payable quarterly in arrears for 4 years, at a rate of interest of 12% per annum, convertible half-yearly. **[4 marks]**

6. A new issue of a fixed interest security has a term to redemption of 20 years and is redeemable at 110% . The security pays a coupon of $9\frac{1}{2}$ per annum, payable half-yearly in arrears.

An investor who is liable to tax on all income at a rate of 23% and on all capital gains at a rate of 34% bought all the stock at the date of issue at a price which gave the investor a yield to maturity of 8% per annum effective.

What price did the investor pay per \$100 nominal of the stock. **[7 marks]**

7. The current price of an asset is \$10 000 and the risk-free force of interest is 5% per annum.

Calculate the forward price for this asset payable in 9 months' time assuming:

- (a) No dividends are payable in this period.
- (b) A dividend of \$600 will be paid in 3 months' time.

[6 marks]

8.(i) Explain briefly the concept of immunization in the context of the assets and liabilities of a life office and state three conditions necessary for applying Redington's theory of immunizations. **[6 marks]**

(ii) An insurance company has issued a number of policies for which a total liability of \$1 000 000 is payable in exactly 10 years' time. The insurance company holds sufficient funds to cover the liability on the basis of a force of interest of δ per annum and intends to invest this money solely in the purchase of the following types of bond:

- A : Zero-coupon bonds redeemable at par in 20 years' time
- B : Zero-coupon bonds redeemable at part in 5 years' time

What proportion of the company's funds should be invested in the 5 year zero-coupon bonds to ensure that the discounted term of the assets equals the discounted mean term of the liabilities at a constant force of interest δ per annum given that this is also the market basis for pricing all stocks? **[9 marks]**

[Total : 15 marks]

9. Let i_t denote the interest rate in year t . It is assumed that for each year t , i_t will be

$$i_t \begin{cases} 0.03 & \text{with probability } \frac{1}{3} \\ 0.07 & \text{with probability } \frac{2}{3} \end{cases}$$

Calculate the expected value and the standard deviation $(1 + i_t)$. **[6 marks]**

10. The following data relates to the assets of a small pension fund:

Date	Market Value
1 January 1999	25 000
1 April 1999	29 000
1 July 1999	30 000
1 October 1999	32 000
1 January 2000	31 000

The only cashflow during 1999 that was not generated from the assets of the fund was an injection of \$5 000 on 31st March 1999.

Calculate for the year 1999:

- (i) The money-weighted rate of return **[3 marks]**
 - (ii) The time-weighted rate of return **[3 marks]**
 - (iii) The linked quarterly rate of return **[3 marks]**
- [Total:9 marks]**

11. The force of interest $\delta(t)$ is a function of time, t (measured in years) and at any time t is given by the formula

$$\delta(t) = \frac{0.05}{1 + 0.05t}$$

- (i) Obtain the present value (as a function of t) of 1 unit due at time t . **[4 marks]**
 - (ii) Evaluate $(\bar{Ia})_{\overline{10}|}$, where the present value is evaluated at time $t = 0$. **[6 marks]**
- [Total : 10 marks]**

12. The n -year spot rate of interest, y_n is given by:

$$y_n = 0.04 + \frac{n}{1000} \quad \text{for } n = 1, 2 \text{ and } 3$$

- (i) Calculate the implied one-year forward rates applicable at time $t = 1$ and $t = 2$. **[4 marks]**
 - (ii) Assuming that coupon and capital payments may be discounted using the same discount factors, and that no arbitrage applies, calculate:
 - (a) The price at time $t = 0$ per \$100 nominal of a bond which pays annual coupons of 3% in arrears and is redeemed at 110% after 3 years. **[4 marks]**
 - (b) The 2-year par yield **[3 marks]**
- [Total: 11 marks]**

END OF EXAMINATION PAPER!!!