NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (ACTUARIAL SCIENCE) HONOURS DEGREE

ACTUARIAL MATHEMATICS I – CIN 2110

NOVEMBER/DECEMBER 2005 EXAMINATION

DURATION: 3 HOURS

Instructions To Candidates

1. *Attempt ALL 13 questions, beginning each question on a new sheet.*
2. *For this question paper you are permitted to have an electronic calculator (non – programmable) and actuarial tables*
3. *You must not start writing your answers until instructed to do so by the invigilator*
4. *Mark allocations are shown in brackets*

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1. Describe what is meant by a 'currency swap'. **[2 marks]**

2. A certain company issues both debenture stock and unsecured loan stocks. Describe the differences between these two types of asset. **[4 marks]**

3. For a rate of interest 7% per annum, convertible monthly calculate:

   (i) The equivalent rate of interest per annum convertible half yearly.
   [2 marks]
   (ii) The equivalent rate of discount per annum convertible monthly.
   [2marks]
   [Total: 4 marks]

4. You are given the following information in respect of a pension fund.

Calender Year	*Value of fund at 01 Jan*	*Value of fund at 30 June*	*Net cash flow received 01 July*
1997	180 000	212 000	25 000
1998	261 000	230 000	18 000
1999	273 000	295 000	16 000
2000	309 000		

Calculate the annual effective time weighted rate of return earned on the fund over the period from 01 January 1997 to 01 January 2000. **[5 marks].**

5. A government issues a 90-day treasury bill at a simple rate of discount of 5% per annum. Calculate the rate of return per annum convertible half yearly received by an investor. **[3 marks]**

6. An investor is considering the purchase of 100 ordinary shares in a company. Dividends from the share will be paid annually. The next dividend is due in one year and is expected to be $8 per share. The second dividend is expected to be 8% greater than the first dividend and the third dividend is expected to be 7% greater than the second dividend. Thereafter dividends are expected to grow at 5% per annum compound in perpetuity.

   Calculate the present value of this dividend stream at a rate of interest of 7% per annum effective. **[5 marks]**

7. The n-year spot rate of interest Y_n is given by:

   $$Y_n = 0.04 + \frac{n}{1000}$$

   (i) Calculate the implied one year forward rates applicable at times $t = 1$ and $t = 2$.
   (ii) Assuming that coupon and capital payments may be discounted using the same discount factors and that no arbitrage applies, calculate:
   (a) The price at $t = 0$ per 100 nominal of a bond which pays annual coupons at 3% in arrears and redeemed at 110% after 3 years.
   [6 marks]
   (b) The 2 – year par yield. **[3 marks]**
   [Total: 9 marks]

8. In any year, the rate of interest on funds invested with a given insurance company is independent of the rates of interest in all previous years.

   Each year the value of $(1 + i_t)$, where i_t is the rate of interest earned in the tth year, is log normally distributed. The mean and standard deviation of i_t are 0.07 and 0.20 respectively.

   (i) Determine the parameters μ and δ^2 of the lognormal distribution of $(1 + i_t)$.
   [5 marks]
   (ii) (a) Determine the distribution of S_{15}, where S_{15} denotes the accumulation of one unit of money invested over 15 years.

(b)   Determine the probability the $S_{15} > 2.5$.   **[4 marks]**

[Total: 9 marks]

9.   A loan of nominal amount $100 000 is to be issued bearing interest payable half –
yearly in arrears at a rate of 7% per annum. The loan redeemable with a capital
payment of 110 per $100 nominal on a coupon date between 10 and 15 years after
the date of issue, inclusive, the date of redemption being at the option of the
borrower.

An investor who is liable to income tax at 25% and capital gains tax of 30%
wishes to purchase the entire loan at the date of issue at a price which ensures that
the investor achieves a net effective yield of at least 5% per annum.

(i)   Determine whether the investor would make a capital gain if the
investment is held until redemption.   **[3 marks]**

(ii)   Explain how your answer to (i) influences the assumptions made in
calculating the price the investor should pay.   **[2 marks]**

(iii)   Calculate the maximum price which the investor should pay. **[5 marks]**

[Total: 10 marks]

10.   (i)   Explain what is meant by no-arbitrage assumption in financial
mathematics.   **[2 marks]**

(ii)   A 7-month forward contract is issued on 01 January 2000 on stock with a
price of $60 per share. Dividends of $2 per share are expected after 3
months and 6 months.
Assuming a risk free force of interest of 7% per annum and no arbitrage,
Calculate the forward price.   **[4 marks]**

[Total: 6 marks]

11.   A loan of $ 80 000 is payable over 25 years by level monthly instalments in
arrears of capital and interests. The repayments are calculated using an effective
rate of interest of 8% per annum, calculate:

(i)   (a)   the capital repaid in the first monthly instalment.
(b)   the total amount of interest paid during the last six years of the
loan.
(c)   The interest included in the final monthly payment. **[9 marks]**

(ii)   Explain how your answer to (i b) would alter if under the original terms,
of loan repayment had been made less frequently than monthly.

[3 marks]
[Total: 12 marks]

12. The risk free force of interest $\delta(t)$ at time t is given by :

$$\delta(t) = 0.05 \qquad \text{for} < t \leq 10$$
$$= 0.08 + 0.003\ t \qquad t > 10$$

   (i)   (a)   Calculate the accumulation at time 15 of $100 invested at time $t = 5$.

          (b)   Calculate the accumulation at time 14 of $100 invested at time $t = 5$.

          (c) Calculate the accumulation at time 15 of $100 invested at time 14.

          (d)   Calculate the equivalent constant force of interest from $t = 5$ to $t = 15$.   **[9 marks]**

   (ii)   Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $100\ e^{0.01t}$ from $t = 0$ to time $t = 15$.
                                                                    [4 marks]
                                                          [Total: 13 marks]

13. A pension fund expects to make payments of $100 000 per annum at the end of each of the next five years. It wishes to immunize these Liabilities by investing in two zero coupon bonds which, mature in five years and in one year respectively. The rate of interest is 5% per annum effective.

   (i)   (a)   Show that present value of liabilities is $432 948
          (b)   Show that the duration of Liabilities is 2.9 years.   **[6 marks]**

   (ii)   Calculate the nominal amounts of the two zero coupon bonds which must be purchased if the pension fund is to equate the present value and duration of assets and Liabilities.   **[6 marks]**

   (iii) (a)   Calculate convexity of the assets.
          (b)   Without calculating the convexity of the liabilities, comment on whether you think Redding ton's immunization has been achieved.
                                                                    [6 marks]
                                                          [Total: 18 marks]

END OF EXAMINATION