

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (ACTUARIAL SCIENCE) HONOURS DEGREE

ACTUARIAL MATHEMATICS I – CIN 2110

JULY/AUGUST SUPPLEMENTARY 2006 EXAMINATION

DURATION: 3 HOURS

Instructions to Candidates

1. Attempt ALL 11 questions, beginning each question on a new sheet.
 2. For this question paper you are permitted to have an electronic calculator (non – programmable) and actuarial tables
 3. You must not start writing your answers until instructed to do so by the invigilator
 4. Mark allocations are shown in brackets
-

1. An individual purchases \$100 000 nominal of a bond on 01 January 2003 which is redeemable at 105 in four years time and pays coupons of 4% per annum at the end of each year.

The investment manager wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year.

Deriving the necessary formulae, calculate the mean value of the total accumulated investment on 31 December 2006 if the annual effective rate of interest has an expected value of 5 1/2 % in 2004, 6% in 2005 and 4 1/2 % in 2006. **[5 marks]**

2. The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.05 & 0 < t < 8 \\ 0.04 + 0.0004t^2 & 8 \leq t \leq 15 \end{cases}$$

Calculate the accumulated value at time $t = 15$ of a continuous payment stream of \$50 per annum payable from $t = 0$ to $t = 8$. **[5 marks]**

3. (i) Calculate the present value of \$100 over ten years at the following rates of interest/discount.
- (a) a rate of interest of 5% per annum convertible monthly
 (b) a rate of discount of 5% per annum convertible monthly.
 (c) a force of interest of 5% per annum. **[6 marks]**
- (ii) A 91-day treasury bill is bought for \$98.91 and is redeemed at \$100. Calculate the annual effective rate of interest obtained from the bill. **[2 marks]**
[Total: 8 marks]
4. (i) State the features of a Eurobond. **[3 marks]**
- (ii) An investor purchases a Eurobond on the date of issue at a price of \$97 per \$100 nominal. Coupons are paid annually in arrear. The bond will be redeemed at par twenty years from the issue date. The rate of return from the bond is 5% per annum effective.
- (a) Calculate the annual rate of coupon paid by the bond.
 (b) Calculate the duration of the bond. **[6 marks]**
[Total: 9 marks]
5. An investor is considering two investment projects A and B. Both involve outlays of \$1 million. Project A will provide a single incoming cash payment after 8 years of \$1.7 million. Project B will provide incoming cash payments of \$1 million after 8 years, \$0.321 million after 9 years, \$0.229 million after 10 years and \$0.245 million after 11 years.
- (a) Determine the rate of interest (i) at which the net present value of the two projects is equal. **[2 marks]**
- (b) By general reasoning or by illustrative calculation, show that at a positive rate of interest i^* where $i^* < i$, project B will have a higher net present value than project A. **[6 marks]**
[Total: 8 marks]
6. A fixed interest security was issued on 01 January in a given year. The security pays half-yearly coupons of 4% per annum. The security is redeemable at 110% 20 years after issue. An investor who pays both income and tax and capital gains tax at a rate of 25% buys the security on the date of issue. Income tax is paid on coupons at the end of the calendar year in which the coupon is received. Capital gains tax is paid immediately on sale or redemption.
- (i) Calculate the price paid by the investor to give a net rate of return of 6% per annum effective. **[5 marks]**

- (ii) Calculate the duration of the net payments from the fixed interest security for an investor who pays income tax as described above but who does not pay capital gains tax, at a rate of interest 6% per annum effective.

[6 marks]

[Total: 11 marks]

7. An ordinary share pays annual dividends. The next dividend is expected to be \$0.05 per share and is due in exactly 3 months time. It is expected that subsequent dividends will grow at a rate of 4% per annum compound and that inflation will be 1.5% per annum. The price of the share is \$1,25 and dividends are expected to continue in perpetuity.

Calculate the effective real rate of return per annum for an investor who purchases the share.

[7 marks]

8. A bank makes a loan of \$100 000 to an individual. The loan is to be repaid by level monthly instalments in arrears over a period of 25 years. The instalments are such that the borrower pays interest at an effective rate of 5% per annum on the loan.

- (i) Calculate the amount of the instalments. **[4 marks]**

- (ii) (a) Calculate the capital outstanding after 12 years, immediately after payment of the instalment then due.

- (b) Determine the split of the 145th instalment between capital and interest. **[6 marks]**

[Total: 10 marks]

9. In any year t , the yield on a fund of investments has mean j_t and standard deviation s_t . In any year, the yield is independent of the value in any year. The accumulated value, after n years, of a unit sum of money invested at time 0 is S_n .

- (i) Derive formulae for the mean and variance of S_n if $j_t = j$ and $S_t = s$ for all years t . **[5 marks]**

- (ii) (a) Calculate the expected value S_8 if $j = 0.06$

- (b) Calculate the standard deviation of S_8 if $j = 0.06$ and $s = 0.08$.

[4 marks]

[Total: 9 marks]

10. An insurance company has a portfolio of annuity contracts under which it expects to pay \$1 million at the end of each of the next 20 years, followed by \$0,5 million at the end of each of the following 20 years. The government bond with the longest duration in which it can invest its funds pay a coupon of 10% per annum in arrears and is redeemed at par in 15 years time. The yield to maturity of the government bond is 6% per annum effective and a coupon payment has just been made.

- (i) (a) Calculate the duration of the insurance company's liabilities at a rate interest of 6% per annum effective.
- (b) Calculate the duration of the insurance company's assets at a rate interest of 6% per annum effective, if all the insurance company's funds are invested in the government bond with the longest duration. **[8 marks]**
- (ii) (a) Explain why the insurance company cannot immunize its liabilities by purchasing government bonds.
- (b) Without any further calculations, state the circumstances under which the insurance company would make a loss if there were a uniform change in interest rates. Explain why a loss would be made. **[5 marks]**
- [Total: 13 marks]**
11. (i) (a) Explain what is meant by the "expectations theory" explanation for the shape of the yield curve.
- (b) Explain how expectations theory can be modified by both liquidity preference and market segmentation theories. **[6 marks]**
- (ii) Short term, one – year annual effective interest rates are currently 10%; they are expected to be 9% in one year's time, 8% in two year's time, 7% in three years' time and to remain at that level thereafter indefinitely.
- (a) If bond yields over all terms to maturity are assumed only to reflect expectations of the future short – term interest rates, calculate the gross redemption yields from 1-year, 3-year, 5-year and 10 – year zero coupon bonds.
- (b) Draw a rough plot of the yield curve for zero coupon bonds using the data from part (ii)(a). (Graph paper not required).
- (c) Explain why the gross redemption yield curve for coupon paying bonds will slope down with a less steep gradient than the zero coupon bond yield curve. **[9 marks]**
- [Total: 15 marks]**

END OF EXAMINATION