

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONS) IN ACTUARIAL SCIENCE

APRIL/ MAY 2009 EXAMINATIONS

ACTUARIAL MATHEMATICS 1 (CIN 2110)

TIME ALLOWED: 3 HOURS

INSTRUCTIONS TO CANDIDATES

- 1 Answer all 10 questions.
- 2 Write clearly and begin each question on a new page.
- 3 In addition to this paper a candidate should have a **copy of actuarial tables** and a non-programmable scientific calculator.

Q1 Show that $(Ia)_n = \frac{\ddot{a}_n - nv^n}{i}$ [5 marks]

[Total 5 marks]

Q2 A loan of \$ 10 000 is to be repaid over ten years by a level annuity payable monthly in arrear. The amount of the monthly payment is calculated on the basis of an interest rate of 1% per month effective. Find:

- (a) The monthly repayment. [2 marks]
- (b) The total capital repaid and interest paid in, (i) the first year and (ii) the final year. [4 marks]
- (c) After which monthly repayment the outstanding loan is first less than \$ 5000. [3 marks]
- (d) For which monthly repayment the capital repaid first exceeds the interest content. [3 marks]

[Total 12 marks]

Q3 Assume that $\delta(t)$, the force of interest per annum at time t (years), is given by the formula:

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t < 5 \\ 0.06 & \text{for } 5 \leq t < 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

- (a) Derive expressions for $v(t)$, the present value of 1 due at time t . [3 marks]
- (b) An investor effects a contract under which he will pay 15 premiums annually in advance into an account which will accumulate according to the above force of interest. Each premium will be of amount \$ 600 and the first premium will be paid at time 0. In return the investor will receive either:
- (i) The accumulated amount of the account one year after the final premium is paid, or
- (ii) A level annuity payable annually for eight years, the first payment being made one year after the final premium is paid.
- Find the lump sum payment under option (i) and the amount of the annual annuity under option (ii). [8 marks]

[Total 11 marks]

Q4 A company is considering two capital investment projects. Project A requires an immediate expenditure of \$ 1 000 000 and will produce returns of \$ 270 000 at the end of each of the next eight years. Project B requires an immediate investment of \$ 1 200 000 together with further expenditure of \$ 20 000 at the end of each of the first three years, and will produce returns of \$ 1 350 000 at the end of the sixth, seventh, and eighth years.

- (a) Calculate (to the nearest 0.1%) the internal rate of return per annum for each project. [4 marks]
- (b) Find the net present value of each project on the basis of an effective annual interest rate of 15%. [4 marks]
- (c) Comment briefly on yours answers. [2 marks]

[Total 10 marks]

Q5 The table below shows the progress of a lottery winner's investment portfolio for the 2004 calendar year:

	Fund Value	Cash Flow
1 January	\$ 75 000	
31 March	\$ 90 000	
1 April	Won Lottery. Winnings invested:	\$ 2 700 000
30 September	\$ 2 600 000	
1 October	Funds withdrawn to pay off gambling debt:	\$ 2 500 000
31 December	\$ 125 000	

Calculate (to the nearest %) the Money Weighted Rate of Return and the Time Weighted Rate of Return for 2004 for this portfolio and comment on your answers. [Total 5 marks]

- Q6** (i) Define arbitrage and explain why arbitrage may be considered impossible in many markets. [4 marks]

An investor wishes to enter into a forward contract to buy some shares in Company X, maturing in 10 year's time. The current share price is \$ 2,50 per share and the current dividend is 8p per annum. The risk free rate of interest is assumed to be 4% per annum for the next five years and 5% per annum for the following five years.

- (ii) Calculate the forward price of the contract assuming a constant dividend yield and assuming that dividends are paid continuously. [5 marks]

(iii) Determine the value of the forward contract after 6 years, when the share price is \$ 2,90, assuming that the risk-free rate of interest for the remaining term is 5% per annum. Explain the reasoning behind any formula that you use. [6]
[Total 15 marks]

- Q7** (a) At time $t = 0$, the two- year spot rate is 4% per annum effective, the three- year spot rate is 5% per annum effective and the four-year spot rate is 6% per annum effective. Calculate the two-year continuous time forward rate from time $t = 2$. [2 marks]

- (c) At 1 July 2004, an investor has a liability of \$ 20 000 to be paid on 1 January 2008 and a liability of \$ 18 000 to be paid on 1 July 2010. The investor currently holds assets with a present value equal to the present value of the liabilities.
The investor wishes to immunize its portion by investing in two zero coupon bonds with outstanding terms of four years and seven years. Determine whether or not this is possible assuming an effective interest rate of 10% per annum. [8 marks]

[Total 10 marks]

- Q8** An investor purchases a bond three months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation).

- (i) Calculate the purchase price of the bond per \$100 nominal to provide the investor with a rate of return of 8% per annum effective. [6 marks]

- (ii) The real rate of return expected by the investor from the bond is 3% per annum effective. Calculate the annual rate of inflation expected by the investor. [2 marks]

[Total 8 marks]

Q 9 The force of interest $\delta(t)$ is:

$$\delta(t) = 0.005t + 0.0001t^2 \text{ for all } t.$$

- (i) At $t = 8$, calculate the accumulated value of an investment of \$ 100 made at time $t = 0$. [3 marks]

- (ii) Calculate the constant annual effective rate of interest over the eight year period. [2 marks]

[Total 5 marks]

Q10 A company is adopting a particular investment strategy such that the expected annual rate of return from investment is 7% and the standard deviation of the annual returns is 9%. Annual returns are independent and $(1 + i_t)$ is lognormally distributed where i_t is the return in the t^{th} year. The company has received a premium of \$ 1000 and will pay the policyholder \$ 1 400 after 10 years.

- (i) Calculate the expected value and standard deviation of an investment of \$ 1000 over 10 years, deriving all formulae that you use. [8 marks]

- (ii) Calculate the probability that the accumulation of the investment will be less than 50% of its expected value in ten year's time. [8 marks]

- (iii) The company has invested \$ 1 200 to meet its liability in 10 year's time. Calculate the probability that it will have insufficient funds to meet its liability. [3 marks]

[Total 19 marks]

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