NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLGY

BACHELOR OF COMMERCE DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL STATISTICS 1 CIN (2111)

FIRST SEMESTER EXAMINATION : DECEMBER 2002

Instructions

- Answer all questions
- In addition to this paper you should have available Actuarial Tables , Graph paper and electronic calculator.

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1. The movement of Juru stock price is modelled as follows;

In each time period, the stock either goes up with probability 0.35 , stays the same with probability 0.35 , or goes down with probability 0.3

The change in price after 500 time periods is being considered.

i. Assuming that the changes in the successive time periods are independent, explain why the normal distribution can be used as approximate model.

ii. Calculate the approximate value of the probability that, after 500 time periods, the stock will be up by more than 20 from where it started, assuming that changes in successive time intervals are independent. **[5 marks]**

2 A random sample of insurance policies on the contents of private houses written by each of the three companies was examined, and the sum insured (a) under each of the policy noted. The results are summarised below (in units of $1000)

Company *i*	Sample size n_i	Sample mean $\bar{a}$	Sample Variance S^2_i
1	15	115.13	196.41
2	10	95	341.33
3	12	135.42	609.36

Consider the model

$A_{ij} = \mu + \tau_i + e_{ij}$ $\quad$ $i = 1,2,3 \quad j = 1,2,\ldots\ldots\ldots\ldots. n_i.$

Where A_{ij} is the j th sum insured for company i, n_i is the number of responses available for the company i , the e_{ij} are the independent normal variables, each with mean zero and variance σ^2 and $\sum_{i=1}^{3} n_i \tau_i = 0$

i. Calculate the values of the least squares/ maximum likelihood estimates of μ and τ_i .i =1,2,3

ii. Perform an analysis of variance to investigate whether real differences exist among the means of the sums insured under such policies issued by the three companies. **[8 marks]**

3 Consider a random sample of 47 white-collar workers and a random sample of 24 blue-collar workers from the workforce of a large company. The mean salary for the sample of white-collar workers is £28,470 and the standard deviation is £4,270; whereas the mean salary for the sample of blue-collar workers is £21,420 and the standard deviation is £3,020.

Calculate the mean and the standard deviation of the salaries in the combined sample of 71 employees. **[4 marks]**

4 A market research company intends to estimate the proportion of the population, θ , who support a certain political party. They intend to poll a sample large enough so that a 95% confidence interval for θ has a width of 0.03 or less. It is thought that θ is approximately equal to 0.4.

Assuming that everyone questioned will respond to the poll, calculate the minimum size of sample which the company should take.

[4 marks]

5 Let Z be a random variable with mean 0 and variance 1, and let X be a random variable independent of Z with mean 5 and variance 4. Let $Y = X - Z$.

Calculate the correlation coefficient between X and Y. **[3 marks]**

6 Suppose that a random sample of nine observations is taken from a normal distribution with mean ③=0. Let $\bar{U}$ and S^2 denote the sample mean and variance respectively.

Determine to two decimal places the probability that $\bar{U}$ exceeds the value of S.

[3 marks]

7 In an investigation into the proportion (p) of the lapses in the first year of a certain type of policy, the uncertainty about p is modelled by taking p to beta to have a beta distribution with parameters $\alpha=1$ and $\beta = 9$, that is with density

$f(p) = 9(1-p)^8$

Using this distribution, calculate the probability that p exceeds 0.2

[3 marks]

8 Under a particular model for the evolution of the size of a population over the time , the probability generating function of X_t , the size at time t, $G_x(s)$, is given by

$G_x(s) =\{(s+\lambda t(1-s))/(1+\lambda t(1-s))\}$ where $G_x(s) = E(s^X)$

If the population dies out, it remains extinct for ever.

(a) Show that the expected size of the population at any time t is 1.

(b) Show that the probability that the population has become extinct by the time t is given by $\lambda t(1+\lambda t)$

(c) Comment briefly on the future prospects for the population. **[6 marks]**

9 An insurance company issues house buildings policies for houses of similar size in four different post-code regions A, B, C and D.

(i) An insurance agent takes independent random samples of 10 house buildings policies for houses of similar size in regions A and B. The annual premiums (£) were as follows:

Region A: 229 241 270 256 241 247 261 243 272 219

($\sum x$ = 2,479 ; $\sum x^2$ = 617,163)

Region B: 261 269 284 268 249 255 237 270 269 257

($\sum x$ = 2,619 ; $\sum x^2$ = 687,467)

(a) Perform a two-sample t-test at the 5% level to compare the premiums for these two regions.

(b) Present the data in a simple diagram and hence comment briefly on the validity of the assumptions required for the above t-test.

(c) Calculate a 95% confidence interval for the underlying common standard deviation σ of such premiums.

[10 marks]

(ii) The agent takes further independent random samples of 10 such policies from the other two regions C and D. The annual premiums were as follows:

Region C: 253 247 244 245 221 229 245 256 232 269

($\sum x$ = 2,441 ; $\sum x^2$ = 597,607)

Region D: 279 268 290 245 281 262 287 257 262 246

($\sum x$ = 2,677 ; $\sum x^2$ = 718,973)

(a) Perform a one-way analysis of variance at the 5% level to compare the premiums for all four regions.

(b) Present the new data in a simple diagram and hence comment briefly on the validity of the assumptions required for the analysis of variance.

(c) Calculate a 95% confidence interval for the underlying common standard deviation σ of such premiums in the four regions. **[10 marks]**

(iii) Comment briefly on your two confidence intervals in (i)(c) and (ii)(c) above.

[1 mark]

[Total 21 marks]

10 An engineer is interested in estimating the probability that a particular electrical component will last at least 12 hours before failing. In order to do this, a random sample of *n* components is tested to destruction and their failure times $x_1, x_2, \ldots\ldots x_n$ are recorded. The engineer models failure times by assuming that they come from a distribution with distribution function, *F*, and probability density function, *f*, given below.

$F(x) = 1- 1/(1+x)^{\alpha-1}$ $f(x) = (\alpha - 1)/(1+x)^{\alpha-1}$ $\alpha > 1, \; x > 1$

(i) Determine the maximum likelihood estimator of α, and, assuming *n* is large, use asymptotic theory to show that an approximate 95% confidence interval for α is given by

$\alpha \pm 1.96 (\alpha-1)/\sqrt{n}$ **[8 marks]**

(ii) A sample of size $n = 80$ leads to a maximum likelihood estimate of α of 1.56. Use this figure to:

(a) estimate the probability a component will fail before 12 hours,

(b) determine an approximate upper 95% one-sided confidence interval for α, and

(c) hence determine an approximate 95% one-sided confidence interval which provides an upper bound for the probability in part (ii)(a) above.

[6 marks]

(iii) Sixty-one of the eighty components tested in part (ii) failed before 12 hours, so a second engineer estimates the failure probability by 61/80 = 0.7625, and constructs an upper 95% confidence interval based on the binomial distribution.

(a) Construct this interval, and

(b) comment on the advantages and disadvantages of this method when compared to the method of part (ii).

[4 marks]

[Total 18 marks]

11 A random sample of 200 unrelated motor policies with identical risk profiles from a certain company's business gave rise to a total of 52 claims in 1996. Note that each policy can give rise to more than one claim in any one year.

Assuming a Poisson model with mean λ for the number of claims made on such a policy in 1996, use a normal approximation to calculate a lower 95% confidence interval for λ .

[5 marks]

12 The number of milligrams of tar in random samples of filter and non-filter cigarettes were recorded as follows:

Filter 0.9 1.1 0.1 0.7 0.3 0.9 0.8 1.0 0.4

$\sum x = 6.2$, $\sum x^2 = 5.22$

Non-filter 1.5 0.9 1.6 0.5 1.4 1.9 1.0 1.2 1.3 1.6 2.1

$\sum x = 15.0$, $\sum x^2 = 22.54$

By making suitable assumptions which should be stated, find a 90% confidence interval for the ratio of the underlying variances in the amounts of tar in the two types of cigarettes. Comment briefly on your result as regards the hypothesis of equal variances.

[5 marks]

13 A random sample of n claim sizes arising under a group of policies is examined. However, the data are censored at a known amount c. There are m claim sizes lower than c (the values of which are known) and $n - m$ higher than c (the values of which are not known).

The known data are therefore n, m and the observed claim sizes x_1 , x_2 , ..., x_m .

Suppose the claim sizes are exponentially distributed random variables with cumulative distribution function

$F(x) = 1 - exp(-\lambda x)$, $x > 0$ (= 0 otherwise)

(i) (a) Show, giving a clear explanation, that the likelihood function of these data is given by:

$$L = \lambda^m \exp[-\lambda\left\{\sum_{i=1}^{m} x_i + (n-m)c\right\}$$

(b) Show that the maximum likelihood estimate of λ is given by

$$\hat{\lambda} = \frac{m}{\sum_{i=1}^{m} x_i + (n-m)c}$$

(c) State the formula for the maximum likelihood estimate of the mean claim size $\mu = 1/\lambda$ and comment briefly on the result. [7]

(ii) In a particular case with c = £10,000, the observed data are $n = 200$, $m = 173$, and

$$\sum_{i=1}^{173} x_i = 638{,}327$$

(a) Calculate the maximum likelihood estimates of λ and of the mean claim size.

(b) Find the estimated asymptotic standard error of $ λ , and hence calculate an approximate 95% confidence interval for the mean claim size.

[8 marks]

[Total 15 marks]

END OF EXAMINATION!!