NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B. COMM (HONOURS) DEGREE ACTUARIAL SCIENCE

ACTUARIAL STATISTICS I : CIN 2111

NOVEMBER/DECEMBER 2004 FIRST SEMESTER EXAMINATION

DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Candidates should answer all questions.
2. In addition to this paper candidates should have a copy of statistical tables and an electronic calculator.

Question 1

A committee consists of 7 men and 4 women, and a sub-committee of 6 is to be chosen at random. Find the probability that the sub-committee contains exactly k women, k = 0, 1, 2, 3, 4. **[4 marks]**

Question 2

A bag contains 4 white and 3 red balls. Two balls are drawn out at random without replacement.

(i) What is the probability that they are white and red respectively? **[1 mark]**

(ii) What is the probability that the second ball drawn is white? **[2 marks]**

[Total 3 marks]

Question 3

If the probability density of X is given by

$$f(x) = \begin{cases} 6x(1-x) & \text{for } 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability density of $Y = X^3$ **[4 marks]**

Question 4

The number of claims arising in a period of one month from a group of policies can be modeled by a Poison distribution with mean 24. Determine the probability that fewer than 20 claims arise in a particular month

[2 marks]

Question 5

Let (X, Y) have joint probability density function f(x,y) = $e^{-(x+y)}$, for x > 0, and y > 0

Find:

(a) P(X < Y) **[2 marks]**

(b) P(X+ Y < 1) **[3 marks]**.

(c) the distribution function F(x, y) **[3 marks]**
[Total 8 marks]

Question 6

Let X be a random variable with probability density function

$$f(x)=\begin{cases} \frac{1}{2}e^{x} & for \quad x\le 0 \\ \frac{1}{2}e^{-x} & for \quad x>0 \end{cases}$$

(a) Show that the moment generating function of X is given by M(t) = $(1 - t^2)^{-1}$ for |t| < 1 **[4 marks]**

(b) Hence find the mean and standard deviation of X. **[2, 3 marks]**
[Total 9 marks]

Question 7

The number of runs scored by a certain poor batsman in an innings is a random variable X with probability function f(x) = $C2^{-x}$, x = 0, 1, 2,……..

Find:

(a) C **[2 marks]**

(b) the probability generating function of X **[3 marks]**

(c) the mean and variance of X. **[1, 2 marks]**
[Total 8 marks]

Question 8

The size of a claim, X, which arises under a certain type of insurance contract, is to be modeled using a gamma random variable with parameters α and θ (both > 0)such that the moment generating function of X is given by:

$$M(t) = (1-\theta t)^{-\alpha} \quad \text{for} \quad t < 1/\theta$$

By using the cumulant generating function of X, or otherwise, show that the coefficient of skewness of the distribution of X is given by $2/\sqrt{\alpha}$ **[5 marks]**

Question 9

Let Z be a random variable with mean 0 and variance 1, and let X be a random variable independent of Z with mean 5 and variance 4. Let Y = X - Z.
Calculate the correlation coefficient between X and Y. **[5 marks]**

Question 10

There are two boxes which contain balls as follows: Box 1 has 6 white and 4 red, box 2 has 2 white and 8 red. One box is chosen at random and from it a ball is chosen at random. The ball drawn is red.

(i) What is the probability that this ball came from box 1? **[2 marks]**

(ii) What is the probability that a second ball drawn from the same box as the first will also be red? **[3 marks]**

[Total : 5 marks]

Question 11

State fully the Central Limit Theorem. **[5 marks]**

Question 12

A sequence of n independent and identically distributed observations $X_1, X_2, X_3, \ldots X_n$ is taken from the gamma distribution with parameters α and λ.

Consider the random variable Z defined by $Z = \sum_{i=1}^{n} \frac{Yi}{\sqrt{n}}$, where $Yi = \left[\frac{Xi-\mu}{\sigma}\right]$, μ = E(X) and σ^2 = Var (X).

Let the moment generating functions of Z and Y_i be denoted by $M_Z(t)$ and $M_Y(t)$ respectively.

Show that:

(a) $M_Z(t) = \left[M_Y(\frac{t}{\sqrt{n}})\right]^{-\alpha}$ **[4 marks]**

(b) $M_y(t) = \left[(1-\frac{t}{\sqrt{\alpha}})\exp(t/\sqrt{\alpha})\right]^{-\alpha}$ **[5 marks]**

[Total : 9 marks]

Question 13

If the joint probability density of X and Y is given by:

$$f(x,y) = \begin{cases} \frac{2}{7}(x+2y) & for 0<x<1,\ 1<y<2 \\ 0 & elsewhere \end{cases}$$

Find:

(i) $f_X(x)$ **[3 marks]**

(ii) the expected value of $g(X,Y) = \frac{X}{Y^3}$. **[3 marks]**

[Total = 6 marks]

Question 14

Consider three events A, B and C which are such that A and B are mutually exclusive, A and C are independent, and B and C are independent, P(A) = 0.3, P(B) = 0.6, and P(C) 0.5.

Find the probability of the following events:

(a) $A \bigcup (B \bigcap C)$ **[2 marks]**

(b) $(A \bigcup B) \bigcap C$ **[2 marks]**

(c) State with a reason whether or not events A, B and C are independent.
[2 marks]
[Total : 6 marks]

Question 15

The following data are the sizes of the claims ($) for a random sample of 25 recent claims for accidental damage to house fixtures submitted to an insurance company.

612	478	843	669	326	507	1044	592	651	670	765
449	711	1278	532	662	383	746	553	592	881	638
793	419	689								

(a) Construct a stem and leaf diagram as a visual summary of the data. **[3 marks]**

(b) Find the median claim size **[2 marks]**
[Total : 5 marks]

Question 16

Suppose that the joint distribution of two random variables X and Y is as follows:

	Y = 0	Y = 1	Y = 2
X = 0	0	0.25	0
X = 1	0.25	0	0.25
X = 2	0	0.25	0

(a) Determine whether X and Y are correlated **[3 marks]**
(b) Determine whether X and Y are independent **[3 marks]**
[Total : 6 marks]

Question 17

(a) A random sample of size 50 is taken from a population with mean 100 and standard deviation 25. Stating any assumptions used, find (approximately) the probability that the sample mean lies between 95 and 105. **[5 marks]**

(b) A copying machine in a printshop has probability 0.05 of breaking down on any working day. If the machine has no breakdowns during the 5-day working week, a profit of $200 is realized. If 1 or 2 breakdowns occur, the profit is $20, and if 3 or more occur, a loss of $400 is realized. You should assume independence of the likelihood of breakdown from day to day, and that after a breakdown the machine remains shut down for the rest of the day, coming back into service the next day. Find the expected profit realized per working week. **[5 marks]**

[Total: 10 marks]

END OF EXAMINATION PAPER!!!