NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

B.COMM (ACTUARIAL SCIENCE) HONOURS DEGREE

ACTUARIAL STATISTICS I – CIN 2111

NOVEMBER/DECEMBER 2005 FIRST SEMESTER EXAMINATION

DURATION: 3 HOURS

Instructions To Candidates

1. *Attempt ALL 13 questions, beginning each question on a new sheet.*
2. *For this question paper you are permitted to have an electronic calculator (non – programmable) and actuarial tables*
3. *You must not start writing your answers until instructed to do so by the invigilator*
4. *Mark allocations are shown in brackets*
5. *Write clearly and show all workings*

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1. If the probability is 0, 40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it? **[3 marks]**

2. Consider three events A, B, and C which are such that A and B are mutually exclusive, A and C are independent, B and C are independent, $P(A) = 0{,}3$; $P(B) = 0{,}6$ and $P(C) = 0{,}5$

   (a) Find the probability of the events (i) $A \cup (B \cap C)$ and (ii) $(\mathrm{A} \cup \mathrm{B}) \cap C$. **[2; 2 marks]**

   (b) State with a reason whether or not events A, B and C are independent. **[2 marks]**

   [Total: 6 marks]

3. Show that the Variance of a discrete random variable X is given by:

   $$Var\,(\mathrm{X}) = \mathrm{G}^{\backslash\backslash}(1) + G^{\backslash}(1) - \left[G^{\backslash}(1)\right]^2$$

   Where $G(t)$ denotes the probability generating function of X . **[4 marks]**

4. A continuous random variable Y has PDF:

$$f(y) = \begin{cases} y(y-1)(y-2) & 0 \le y \le 2 \\ C & 2 < Y \le 4 \\ O & \text{otherwise} \end{cases}$$

Where C is a constant

(a) Determine the value of C. **[2 marks]**
(b) Find $E[Y]$ **[3 marks]**

[Total: 5 marks]

5. A random variable X that can take values in the range $X > R$, where R is a positive constant, has probability density function Ke^{-2x}, where K is a constant. Derive a formula for the moment generating function of X, stating the values of t for which your formula is valid, and determine the value of the constant K.

[5 marks]

6. A fair coin is tossed repeatedly until 20 heads have been obtained. Calculate approximately the probability that this will require more than 50 tosses.

[6 marks]

7. Bag A contains two tickets, numbered 1 and 2. Bag B contains four tickets, numbered 1, 2, 3 and 4. One ticket is drawn out of each bag at random (and independently). Let X denote the sum of the two numbers on the tickets drawn, and let Y denote the larger of the two numbers (the single number observed if they are equal).

Find the correlation coefficient between X and Y. **[9 marks]**

8. (a) State the central limit theorem. **[4 marks]**

(c) It is assured that the claims arriving at an insurance company per working day has a mean of 40 and a standard deviation of 12. A survey was conducted over 50 working days. Find the probability that the sample mean number of claims arriving per working day was less than 35.

[4 marks]

[Total: 8 marks]

9. If the probability density function of X is given by:

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that $E\left[X^r\right] = \dfrac{2}{(r+1)(r+2)}$ **[4 marks]**

(b) Use this result to evaluate,

$$E\left[(2X+1)^2\right]$$ **[3 marks]**

[Total: 7 marks]

10. (a) The size of a claim, X, which arises under a certain type of insurance contract, is to be modeled using a gamma random variable with parameters α and θ (both > 0) such that the moment generating function of X is given by:

$$M(t) = (1-\theta t)^{-\alpha}, \qquad t < 1/\theta$$

By using the cumulant generating function of X or otherwise, show that the coefficient of skewness of the distribution of X is given by $2/\sqrt{a}$.

[5 marks]

(b) Show that the moment generating function of the Poisson distribution is given by:

$$M_x(t) = e^{\lambda\left(e^t - 1\right)}$$ **[4 marks]**

(c) Hence, or otherwise , show that : Mean = Variance = λ **[4 marks]**

[Total: 13 marks]

11. The joint probability density function of the random variables X and Y is given by :

$$f\left(x, y = X^2 + \frac{XY}{3}\right), \ 0 < \mathrm{x} < 1, \ 0 < \mathrm{y} < 2$$

Find the following:

(a) $F(x, y)$ **[5 marks]**

(b) $P(X + Y < 1)$ **[4 marks]**

(c) $f_x(x)$ **[3 marks]**

(d) $f_y(y)$ **[3 marks]**

(e) $F_x(x)$ **[2 marks]**

(f) $F_y(y)$ **[2 marks]**

(g) Obtain the correlation coefficient between X and Y, $Corr(X,Y)$ **[5 marks]**

[Total: 24 marks]

12. A random variable X has a density function

$$f(x) = \begin{cases} ce^{-3x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Find the following

(a) The value of constant c **[3 marks]**

(b) $P(1 < x < 2)$ **[3 marks]**

[Total: 6 marks]

13. A random sample of fifty claim amounts ($) arising in a particular section of an insurance company's business are displayed below in a stern and leaf plot:

15	14678
16	0233368889
17	0000001233457888
18	3456779
19	0257
20	0
21	3
22	07
23	
24	
25	3
26	
27	3
28	8
29	
30	
31	2

Stern unit = 100
Leaf unit =10

The sum of the fifty amounts (before rounding) is $92780.

Calculate the Mean and Median claim amounts. **[4 marks]**

END OF EXAMINATION