NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

CIN 2111 - ACTUARIAL STATISTICS I/PROBABILITY THEORY

APRIL/MAY 2009-FIRST SEMESTER EXAMINATIONS

DURATION: 3 HOURS

Instructions to Candidates

1. **Attempt all 14 Questions**
2. **To Obtain Full Marks Show ALL appropriate steps to your answers**

Requirements

1. **Actuarial Tables (2002) Edition**
2. **Non-programmable Scientific calculator**

P1. The stem and leaf plot below gives the surrender values (to the nearest 1,000) of 40 endowment policies issued in Uganda and recently purchased by a dealer in such policies in Harare. The stem unit is 10,000 and the leaf unit is 1,000.

```
5 | 4
5 | 7
6 | 13
6 | 688
7 | 0233455
7 | 6677889
8 | 002234555
8 | 678889
9 | 135
9 | 7
```

(i) Determine the median surrender value and the IQR for this batch of policies.

[Total 4 marks]

P2. A bag contains 8 black and 6 white balls. Two balls are drawn out at random, one after the other and without replacement. Calculate the probabilities that:

(i) The second ball drawn out is black. [1]

(ii) The first ball drawn out was white, given that the second ball drawn out is black. [2]

[Total 3 marks]

P3. (a) State and give the general reasoning behind each of the 3 axioms of probability. [5]

(b) Use the axioms of probability to prove the following:

(i) $A \subset B \Rightarrow P(A) \leq P(B)$ and $P(B-A) = P(B) - P(A)$ [3]

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [3]

(iii) Using part (ii) above, what is $P(A \cup B \cup C)$ [1]

[Total 12 marks]

P4. Two integer random variables, *X* and *Y,* have the following joint probability function:

		Y			
		0	1	2	3
Y	1	0.05	0.20	0.15	0.05
	2	0.20	0.15	0.12	0.08

Calculate

(i) E(X) [1]

(ii) E(X¦Y=3) [2]

(iii) Var(Y¦X=2) [3]

[Total 6 marks]

P5. The number of claims arising in one year from a group of policies follows a Poisson distribution with mean 12. The claim sizes independently follow an exponential distribution with mean \$80 and they are independent of the number of claims.
The current financial year has six months remaining.
Calculate the mean and the standard deviation of the total claim amount which arises during this remaining six months.

[Total 4 marks]

P6. A charity issues a large number of certificates each costing \$10 and each being repayable one year after issue. Of these certificates, 1% are randomly selected to receive a prize of \$10 such that they are repaid as \$20. The remaining 99% are repaid at their face value of \$10.

(i) State the Central limit Theorem [4]

(ii)Show that the mean and standard deviation of the sum repaid for a single purchased certificate are \$10.1 and \$0.995 respectively. [2]

Consider a person who purchases 200 of these certificates.

(iii) Calculate approximately the probability that this person is repaid more than \$2,040 by using the Central Limit Theorem applied to the total sum repaid. [3]
(iv) An alternative approach to approximating the probability in (ii) above is based on the number of prize certificates the person is found to hold. This number will follow a binomial distribution.

Use a Poisson approximation to this binomial distribution to approximate the probability that this person is repaid more than £2,040. [3]
(v) Comment briefly on the comparison of the two approximations above given that the exact probability using the binomial distribution is 0.0517. [1]

[Total 13 Marks]

P7. Let (X,Y) be a bivariate r.v. show that: $[E(XY)]^2 \le E(X^2)E(Y^2)$
(*This is known as the Cauchy-Schwarz Inequality*) [3]
Hint: Consider the expression $E[(X-\alpha Y)^2] \ge 0$ for any two r.v.'s X and Y and a real variable α.

[Total 3 Marks]

P8. The total claim amount on a portfolio, S, is modelled as having a compound distribution
$S = X_1 + X_2 + \ldots + X_N$, where N is the number of claims and has a Poisson distribution with mean λ , X_i is the amount of the i^{th} claim, and the X_i *s* are independent and identically distributed and independent of N. Let $M_X(t)$ denote the moment generating function of X_i.

(i) Show, using a conditional expectation argument, that the cumulant generating function of S, $C_S(t)$, is given by: $C_S(t) + \lambda\{M_X(t) - 1\}$
Note: You may quote the moment generating function of a Poisson random variable from the book of Formulae and Tables. [4]
(ii) Calculate the variance of S in the case where λ = 20 and X has mean 20 and variance 10. [2]

[Total 6 Marks]

P9. Let (X, Y, Z) be a trivariate random variable with joint pdf

$$f_{XYZ}(x,y,z) = \begin{cases} ke^{-(ax+by+cz)} & x>0,\ y>0,\ z>0 \\ 0 & \text{otherwise} \end{cases}$$

Where $a,b,c > 0$ and k are constants

(a) Determine the value of k [4]
(b) Find the marginal joint pdf of X and Y, i.e. $f_{XY}(x,y)$ [4]
(c) Find the marginal pdf of X. [4]
(d) Are X, Y and Z independent. [4]
(e) Show that $f_{XYZ}(x,y,z) = f_{Z|X,Y}(z|x,y) f_{Y|X}(y|x) f_X(x)$ [4]

[Total 20 Marks]

P10. An actuary has been advised to use the following positively-skewed claim size distribution as a model for RTA cover claim, with claim sizes measured in units of $100,

$$f(x,\theta)=\frac{x^2}{2\theta^3}\exp\left(-\frac{x}{\theta}\right): \quad 0<x<\infty,\ \theta>0$$

(i) Find the Moments first three non-central moments for the given distribution [6]
(ii)Determine **by integration** the variance of this distribution and calculate the coefficient of skewness. [4]

[Total 10 Marks]

P11. (i) Let $Y=e^X$. Find the pdf of Y if X=$N(\mu;\sigma^2)$ [4]

(ii) Let $\mathbf{X_1, X_2 \ldots X_n}$ be ***n*** independent random variables and $\mathbf{X_i>0}$.

Let $\Lambda = X_1 \cdot X_2 \cdots X_n = \prod_{i=1}^{n} X_i$

Show that for large n, the pdf of Λ is approximately log-normal [4]

[Total 8 Marks]

P12. One variable of interest, T, in the description of a physical process can be modelled as $T = XY$ where X and Y are random variables such that $X \sim N(200, 100)$ and Y depends on X in such a way that $Y|X = x \sim N(x, 1)$.

Simulate two observations of T, using the following pairs of random numbers (observations of a uniform (0, 1) random variable), explaining your method and calculations clearly:

Random numbers
0.5714 , 0.8238
0.3192 , 0.6844

[Total 5 Marks]

P13. Let (X,Y) be a bivariate r.v. with the joint pdf:

$$f_{XY}(x,y)=\frac{x^2+y^2}{4\pi}e^{-(x^2+y^2)/2} \quad -\infty<x<\infty,\ -\infty<y<\infty$$

Show that X and Y are not independent but are uncorrelated.

[Total 5 Marks]

P14. Consider the following three probability statements concerning an F variable with 6 and 12 degrees of freedom. State, with reasons, whether each of these statements is true:

(a) $P(F_{6,12} > 0.250) = 0.95$ [1]
(b) $P(F_{6,12} < 4.821) = 0.99$ [1]
(c) $P(F_{6,12} < 0.130) = 0.01$ [1]

[Total 3 Marks]

*********END OF EXAMINATION*************