NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

CIN 2111 - ACTUARIAL STATISTICS I/PROBABILITY THEORY

AUGUST 2011 SUPPLEMENTARY EXAMINATIONS

DURATION: 3 HOURS

Instructions to Candidates

1. **Attempt all 13 Questions**
2. **To Obtain Full Marks Show ALL appropriate steps to your answers**

Requirements

1. **Actuarial Tables (2002) Edition**
2. **Non-programmable Scientific calculator**

Q1. Calculate three commonly used statistics that provide a measure of spread, and expressed in terms of the original unit of measure, for the following set of ten observations:

5.1 , 2.6 , 7.3 , 4.4 , 4.6 , 2.9 , 3.4 , 3.2 , 4.4 , 5.0 [Total 6marks]

Q2. A bag contains 8 black and 6 white balls. Two balls are drawn out at random, one after the other and without replacement. Calculate the probabilities that:

(i) The second ball drawn out is black. [1]
(ii) The first ball drawn out was white, given that the second ball drawn out is black. [2]

[Total 3 marks]

Q3. (a) Show that, if the random variable X has a Gamma(α,λ) distribution, then X/k, where k is a positive constant, has a Gamma(α,kλ) distribution. [4 marks]

(b) If the random variable T representing the total lifetime of an individual light bulb has an Exp(λ) distribution, where $1/\lambda = 2{,}000$ hours, find the probability that the average lifetime of 10 bulbs will exceed 4,000 hours. [6 marks]

[Total 10 marks]

Q4. Two integer random variables, *X* and *Y*, have the following joint probability function:

		Y			
		0	1	2	3
Y	1	0.05	0.20	0.15	0.05
	2	0.20	0.15	0.12	0.08

Calculate

(i) E(X) [1]

(ii) E(X¦Y=3) [2]

(iii) Var(Y¦X=2) [3]

[Total 6 marks]

Q5. Show from first principles that the variance of the second formulation of the geometric distribution (where x is the number of failures before the first success) with parameter θ is $\frac{1-\theta}{\theta^2}$.

Hint: consider E[X(X+1)] [Total 10 marks]

Q6. A charity issues a large number of certificates each costing $10 and each being repayable one year after issue. Of these certificates, 1% are randomly selected to receive a prize of $10 such that they are repaid as $20. The remaining 99% are repaid at their face value of $10.

(i) State the Central limit Theorem [4]

(ii)Show that the mean and standard deviation of the sum repaid for a single purchased certificate are $10.1 and $0.995 respectively. [2]

Consider a person who purchases 200 of these certificates.

(iii) Calculate approximately the probability that this person is repaid more than $2,040 by using the Central Limit Theorem applied to the total sum repaid. [3]

(iv) An alternative approach to approximating the probability in (ii) above is based on the number of prize certificates the person is found to hold. This number will follow a binomial distribution.

Use a Poisson approximation to this binomial distribution to approximate the probability that this person is repaid more than £2,040. [3]

(v) Comment briefly on the comparison of the two approximations above given that the exact probability using the binomial distribution is 0.0517. [1]

[Total 13 Marks]

Q7. A continuous random variable Y has PDF:

$$f(y) = \begin{cases} y(y-1)(y-2) & 0 \le y \le 2 \\ c & 2 < y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

Determine the value of c and find E[Y].

[Total 5 Marks]

Q8. Show that the variance of a discrete random variable X is given by:

$$\text{Var (X)} = G''(1) + G'(1) - [G'(1)]^2$$

Where G(t) denotes the probability generating function of X.[Total 6 Marks]

Q9. Let (*X, Y, Z*) be a trivariate random variable with joint pdf

$$f_{XYZ}(x, y, z) = \begin{cases} ke^{-(ax+by+cz)} & x>0,\ y>0,\ z>0 \\ 0 & \text{otherwise} \end{cases}$$

Where $a, b, c > 0$ and k are constants

(a) Determine the value of k [4]

(b) Find the marginal joint pdf of X and Y, i.e. $f_{XY}(x, y)$ [4]

(c) Find the marginal pdf of X. [4]

(d) Are X, Y and Z independent. [4]

(e) Show that $f_{XYZ}(x, y, z) = f_{Z|X,Y}(z|x, y) f_{Y|X}(y|x) f_X(x)$ [4]

[Total 20 Marks]

Q10. An actuary has been advised to use the following positively-skewed claim size distribution as a model for RTA cover claim, with claim sizes measured in units of $100,

$$f(x, \theta) = \frac{x^2}{2\theta^3} \exp\left(-\frac{x}{\theta}\right): \quad 0 < x < \infty,\ \theta > 0$$

(i) Find the Moments first three non-central moments for the given distribution [6]

(ii)Determine **by integration** the variance of this distribution and calculate the coefficient of skewness. [4]

[Total 10 Marks]

Q11. Use the cumulant generating functions to find the mean, variance and skewness of the Poisson distribution with parameter μ. [Total 3 Marks]

Q12. Let (X,Y) be a bivariate r.v. with the joint pdf:

$$f_{XY}(x,y) = \frac{x^2 + y^2}{4\pi} e^{-(x^2+y^2)/2} \quad -\infty < x < \infty,\ -\infty < y < \infty$$

Show that X and Y are not independent but are uncorrelated.

[Total 5 Marks]

Q13. Consider the following three probability statements concerning an F variable with 6 and 12 degrees of freedom. State, with reasons, whether each of these statements is true:

(a) $P(F_{6,12} > 0.250) = 0.95$ [1]

(b) $P(F_{6,12} < 4.821) = 0.99$ [1]

(c) $P(F_{6,12} < 0.130) = 0.01$ [1]

[Total 3 Marks]

**********END OF EXAMINATION*************