NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMMERCE

DEPARTMENT OF INSURANCE AND ACTUARIAL SCIENCE

B.COMM (HONOURS) DEGREE IN ACTUARIAL SCIENCE

ACTUARIAL STATISTICS II - CIN 2211

AUGUST 2009 - SECOND SEMESTER EXAMINATIONS

DURATION: 3 HOURS

Instructions to Candidates

1. **Attempt all TWELVE(12) Questions**
2. **To Obtain Full Marks Show ALL appropriate steps to your answers**

Requirements

1. **Actuarial Tables (2002) Edition**
2. **Non-programmable Scientific calculator**
3. **A Graph paper**

P1.

Let ($\mathbf{X_1, X_2\ldots X_n}$) be a random sample from a random variable **X** over (0, t) where **t** is unkown. Show that:

T = Max ($\mathbf{X_1, X_2\ldots X_n}$) is a consistent estimator of the parameter **t**. **[6 marks]**

P2

(a)The time taken to process simple home insurance claims has a mean of 20 minutes and a standard deviation of 5 minutes. Stating clearly any assumptions that you make, calculate the probability that the:

(i) sample mean of 5 claims is less than 15 minutes [2 marks]

(ii) sample variance of 5 claims is greater than 6.65 minutes [2 marks]

(iii) both (i) and (ii) hold [2 marks]

(b) A random sample of 20 observations from $X \sim N(\mu, 8)$ has sample mean $x = 18.6$. Calculate a symmetrical 90% confidence interval for μ. [3 marks]

[Total 9 Marks]

P3

The following test concerning the mean claim amount (μ) for a certain class of policy

H_0: = \$200 vs. H_1 : $\neq$ \$200

is to be performed. A random sample of 18 claims is examined and yields a mean amount of \$212 and standard deviation \$52.

(a) Calculate the approximate probability-value for the test. [3 marks]

(b) Comment on the use of a probability-value compared to a simple 5% test in the above case hence determine the minimum size of a test that will reject H_0. [2 marks]

[Total 5 Marks]

P4

When comparing the mean premiums for policies issued by two companies, a two sample *t* test is performed assuming equal population variances. The sample sizes and sample variances are given by

$n_1 = 25$, $s_1^2 = 139.7$

2

$n_2 = 30$, $s_2 = 76.6$

Perform an appropriate *F* test at the 5% level to investigate the validity of the equal variance assumption. **[3 marks]**

P5

It is known that a random sample of 12,11.2,13.5,12.3,13.8,11.9 comes from a population having the following p.d.f

$$f(X;\theta) = \begin{cases} \dfrac{\theta}{x^{\theta+1}} ; x > 1, \theta > 1 \\ 0 \text{ ; otherwise} \end{cases}$$

Find the Method of Moment Estimator of θ **[3 marks]**

P6

A sample of 12 paired observations (x_i, y_j) yielded a sample correlation coefficient r = 0.91. Use Fisher's transformation to test at the 5% significance level;

H_0: r =0.8 against H_1: r >0.8

where r in the population correlation coefficient.

[5 marks]

P7

For the estimation of a binomial probability $p = P(\text{success})$, a series of n independent trials are performed and X represents the number of successes observed.

(i) Write down the likelihood function $L(p)$ and show that the maximum likelihood estimator (MLE) of p is $\hat{p} = \dfrac{X}{n}$ [3 marks]

(ii)

(a) Determine the Cramer-Rao lower bound for the estimation of p.

(b) Show that the variance of the MLE is equal to the Cramer-Rao lower bound.

(c) Write down an approximate sampling distribution for $\hat{p}$ valid for large n. [4 marks]

(iii) In order to develop an approximate 95% confidence interval for p for large n, the following pivotal quantity is to be used

$$\frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}} \approx N(0,1)$$

Assuming that this pivotal quantity is monotonic in p, show that rearrangement of the inequality

$$-1.96 < \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}} < 1.96$$

leads to a quadratic inequality in p, and hence determine an approximate 95% confidence interval for p of the form $p_L(\hat{p}) < p < p_U(\hat{p})$. [5 marks]

(iv) A simpler and more widely used approximate confidence interval is obtained by using the following pivotal quantity

$$\frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}} \approx N(0,1)$$

Determine the resulting approximate 95% confidence interval using this. [2 marks]

(v) In two separate applications the following data were observed:

(a) 4 successes out of 20 trials
(b) 80 successes out of 200 trials

In each case calculate the two approximate confidence intervals from parts (iii) and (iv) and comment briefly on your answers. [6 marks]

[Total 20 marks]

P8

The number of claims which arise in a year under a policy of a certain type follows a Poisson distribution with mean λ. It is required to test

$H_0 : \lambda = 0.4$ v $H_1 : \lambda > 0.4$

and it is decided to reject H_0 in favour of H_1 if 29 or more claims arise in a year under a group of 50 independent such policies.

(i) Explain the terms:

(a)critical region

(b) Type I error

(c) The power of a test [3 marks]

(ii) Using tables of Poisson distribution probabilities; calculate the power of this test in each of the following cases:

(a) $\lambda = 0.5$, and

(b) $\lambda = 0.6$

(c) Comment briefly on the results. [6 marks]

[Total 9 Marks]

P9

The probabilities of death within one year for the respective partners in a marriage relationship are denoted by q_1 and q_2 and the deaths are assumed to be independent events.

(i) Write down expressions, in terms of q_1 and q_2 , for the probabilities of neither, one only, and both partners dying within one year. [2 marks]

(ii) If, in a random sample of 1000 such couples taken from a population for which $q_1 = 0.1$ and $q_2 = 0.09$, neither partner dies within one year in 825 cases and exactly one partner dies within one year in 159 cases, test the goodness of fit of the above model and comment briefly on the result. [5 marks]

[Total 7 Marks]

P10

An IQ test was administered to 5 persons before and after they were trained. The results are given below:

Candidates	I	II	III	IV	V
IQ before training	110	120	123	132	125
IQ level after training	120	118	125	136	121

Test at 1% whether there is any change in IQ after the training programme.
[$\mathbf{t}_{0.01}(4) = 4$] **[5 marks]**

P11

It is thought that a suitable model for a plumber's charges when called out for a job is a linear one based on a fixed call-out charge and an hourly rate. A random sample of 10 of his invoices gave the following results:

Duration of job (hours) x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.5	5.0	5.5
Cost of job ($) y	40	55	45	65	80	75	95	100	120	130

$\Sigma x = 29$ $\Sigma x^2 = 110.5$ $\Sigma y = 805$ $\Sigma y^2 = 73{,}225$ $\Sigma xy = 2{,}795$

(i)
(a) Plot these data and comment on the suitability of the proposed model.

(b) Calculate the least squares estimates of:

(1) the plumber's call-out charge, and,
(2) the plumber's hourly rate charge. [6 marks]
(ii) (a) Determine a 90% confidence interval for the plumber's hourly rate charge.

(b) Calculate 95% confidence intervals for the expected cost of single jobs lasting

(1) three hours, and
(2) six hours

and comment briefly on their different widths. [9 marks]
[Total 15 Marks]

P12

An Insurance Company issues house building policies for houses of similar size in four different regions A,B,C and D. Independent random samples of 10 policies from each company are examined.

The Annual Premiums (y_{ij} in \$s.) were as follows.

	A	B	C	D
	155	151	145	129
	162	168	131	162
	148	128	155	137
	120	167	172	142
	115	134	141	155
	175	123	175	138
	149	132	155	120
	138	142	162	167
	152	161	186	134
	145	152	148	115
Total	**1459**	**1458**	**1570**	**1399**

$\Sigma y_{ij} = 5886$, $\Sigma y_{ij}^2 = 8{,}78{,}220$

(i) Carry out analysis of variance to examine if there are no significant differences at 5% level among mean premiums charged by each company. [6 marks]

(ii) State the assumptions for the above analysis and drawing suitable diagram of these data, comment briefly on validity of these assumptions. [3 marks]

(iii) Calculate 95% confidence interval for the underlying common standard deviation *s* of such premiums in the four regions. [4 marks]

[Total 13 Marks]

END OF EXAMINATION